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AMERICAN MATHEMATICAL SOCIETY

EDITED BY

EARLE R. HEDRICK

DAVID R. CURTISS WILLIAM R. LONGLEY

WITH THE ASSISTANCE OF

HAROLD T. DAVIS HARRY W. KUHN

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JOHN R. KLINE HENRY L. RIETZ

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THE OCTOBER MEETING IN NEW YORK

The three hundred thirty-fifth meeting of the American Mathematical Society was held at Columbia University on Saturday, October 31, 1936. The attendance included the following one hundred forty members of the Society:

C. R. Adams, H. S. Adams, R. B. Adams, R. L. Anderson, R. C. Archibald, R. G. Archibald, W. L. Ayres, Reinhold Baer, N. H. Ball, M. F. Becker, A. A. Bennett, Garrett Birkhoff, G. D. Birkhoff, Salomon Bochner, H. F. Bohnenblust, C. B. Boyer, A. D. Bradley, H. W. Brinkmann, A. W. Bull, J. Hughes Bushey, J. H. Bushey, J. A. Clarkson, George Comenetz, H. R. Cooley, Richard Courant, D. B. DeLury, Vincent Dillon, Arnold Dresden, Melvin Dresher, W. L. Duren, H. T. Engstrom, Fay Farnum, A. D. Fialkow, E. J. Finan, Edward Fleisher, M. M. Flood, Tomlinson Fort, M. C. Foster, R. M. Foster, J. S. Frame, Philip Franklin, T. C. Fry, D. L. Fuller, A. S. Galbraith, B. P. Gill, S. H. Gould, A. A. Grant, Margaret Gurney, N. L. Haight, Marshall Hall, G. G. Harvey, Max Herzberger, E. H. C. Hildebrandt, Einar Hille, Harold Hotelling, S. E. Hotelling, E. M. Hull, S. A. Joffe, R. A. Johnson, E. R. van Kampen, Edward Kasner, L. S. Kennison, R. B. Kershner, P. W. Ketchum, J. R. Kline, A. W. Landers, M. K. Landers, Solomon Lefschetz, Madeline Levin, E. R. Lorch, A. N. Lowan, R. R. Lyle, J. C. C. McKinsey, E. J. McShane, L. A. MacColl, H. M. MacNeille, H. F. MacNeish, Ingo Maddaus, A. E. Meder, L. T. Moore, T. W. Moore, Richard Morris, Marston Morse, F. J. Murray, D. S. Nathan, C. A. Nelson, John von Neumann, Alta Odoms, Oystein Ore, F. W. Owens, H. B. Owens, H. B. Phillips, E. L. Post, Walter Prenowitz, G. B. Price, R. G. Putnam, J. F. Randolph, H. W. Reddick, M. S. Rees, W. T. Reid, Moses Richardson, J. F. Ritt, M. S. Robertson, S. L. Robinson, Benjamin Rosenbaum, S. G. Roth, Max Sasuly, Stephan Serghiesco, I. M. Sheffer, Jacob Sherman, Max Shiffman, C. G. Shover, L. G. Simons, L. L. Smail, P. A. Smith, R. C. Stauffer, N. E. Steenrod, M. H. Stone, D. J. Struik, J. D. Tamarkin, J. M. Thomas, E. W. Titt, C. B. Tompkins, Annita Tuller, S. M. Ulam, I. L. Van Dyck, Oswald Veblen, H. E. Wahlert, Henry Wallman, Louis Weisner, M. J. Weiss, A. P. Wheeler, Norbert Wiener, W. A. Wilson, H. P. Wirth, W. H. Wise, D. W. Woodard, H. N. Wright, Leo Zippin, H. S. Zuckerman.

The meeting opened on Saturday morning with a session for short papers. In the fore part of the afternoon there was a

general session at which Professor Philip Franklin gave an address on *Transcendental numbers*. Later in the afternoon Professor Henry Norris Russell delivered the thirteenth annual Josiah Willard Gibbs Lecture before a joint meeting with the American Physical Society. His subject was *Model Stars*.

A meeting of the Council was held between the morning and afternoon sessions.

It was reported that the following persons had been elected to membership in the Society:

Professor Oscar Kenneth Bates, The St. Lawrence University, Canton, N. Y.;
Mr. Frank Webster Burton, Fine Hall, Princeton, N. J.;
Professor Élie Joseph Cartan, University of Paris;
Dr. Nicholas Q. Chako, Lynn, Mass.;
Mr. Mortimer James Clingan, New York, N.Y.;
Miss Geraldine Alma Coon, Brown University;
Mr. Robert Victor-Augustus De Vita, Brooklyn, N.Y.;
Dr. Sydney Henry Gould, University of Toronto;
Dr. Ross Arthur Harrison, Oneonta, N.Y.;
Dr. Ivar Edmund Highberg, Whitman College, Walla Walla, Wash.;
Mr. Vern Russel Jones, Tri-State College, Angola, Ind.;
Dr. Bela Adalbert Lengyel, Cambridge, Mass.;
Mr. R. Eric O'Connor, Weston College, Weston, Mass.;
Mr. David Gadalliah Rabinow, Worcester, Mass.;
Dr. Ruth Ramler Struik, Belmont, Mass.;
Mr. John Wilder Tukey, Brown University;
Dr. Lee Roy Wilcox, Institute for Advanced Study.

As nominee of Brown University:

Miss Dorothy L. Bernstein, Brown University.

As nominee of the College of St. Thomas:

Mr. Vernon John Varino, College of St. Thomas, St. Paul, Minn.

The following institutions of learning were elected to institutional membership in the Society:

Acadia University;
Sweet Briar College.

The following appointment by President Lefschetz was announced: as representative of the Society at the inauguration of William Alfred Eddy as president of Hobart College and William Smith College, October 2, 1936, Professor W. B. Carver.

Because of the resignation of Professor G. W. Mullins as Treasurer of the Society and the expiration on October 31, 1936

of the term of office of the Acting-Treasurer, the council elected Professor P. A. Smith to serve as Treasurer for the remainder of the year 1936.

Titles and cross references to the abstracts of the papers read at this meeting follow below; papers whose abstract numbers are followed by the letter *t* were read by title. Papers numbered 1 to 8 were read at the morning session, Professor C. R. Adams presiding; those numbered 9 to 20 were read at the afternoon session, Professor Norbert Wiener presiding. Mr. Morse was introduced by Professor J. R. Kline and Professor Friedrichs by Professor J. D. Tamarkin.

1. *Convergence in variation and related topics*, by Mr. A. P. Morse. (Abstract 42-11-403.)

2. *On the Jacobi condition for the double integral problem of the calculus of variations*, by Dr. W. T. Reid. (Abstract 42-11-406.)

3. *On the extreme points of convex sets*, by Dr. G. B. Price. (Abstract 42-11-405.)

4. *Conformal theorems on triplets of curves*, by Professor Edward Kasner. (Abstract 42-11-402.)

5. *On the behavior of a conformal mapping at a cusp.* II, by Dr. S. E. Warschawski. (Abstract 42-11-409-*t*.)

6. *Conformal geometry on a surface*, by Dr. George Comenetz. (Abstract 42-11-398.)

7. *Initial motion at a point of equilibrium*, by Dr. A. D. Fialkow (National Research Fellow). (Abstract 42-11-400.)

8. *Finite combinatory processes. Formulation* 1, by Dr. E. L. Post. (Abstract 42-11-412.)

9. *Primary abelian groups and their automorphisms*, by Dr. Reinhold Baer. (Abstract 42-11-395-*t*.)

10. *Dualism in abelian groups*, by Dr. Reinhold Baer. (Abstract 42-11-396-*t*.)

11. *Equivalence of algebraic extensions*, by Dr. Reinhold Baer. (Abstract 42-11-397-*t*.)

12. *Schwarzian reflection on a surface*, by Dr. George Comenetz. (Abstract 42-11-399-*t*.)

13. *A simplified set of postulates for a group*, by Mr. R. M. Foster. (Abstract 42-9-329-*t*.)

14. *On some two-dimensional problems in heat conduction*, by Professor A. N. Lowan. (Abstract 42-9-390-*t*.)

15. *On rings of operators.* II, by Dr. F. J. Murray and Professor John von Neumann. (Abstract 42-11-404-*t*.)

16. *A determinant used in functional equations*, by Mr. L. B. Robinson. (Abstract 42-11-407-*t*.)

17. *On the definition of "sense" in the foundations of geometry*, by Dr. A. R. Schweitzer. (Abstract 42-11-408-*t*.)

18. *On the maps of an n-sphere into another n-sphere*, by Professor Hassler Whitney. (Abstract 42-11-410-*t*.)

19. *Cremona webs of surfaces in S_3 without base curves*, by Dr. R. A. Harrison. (Abstract 42-11-401-*t*.)

20. *On certain inequalities and characteristic-value problems for analytic functions and for functions of two variables*, by Professor Kurt Friedrichs. (Abstract 42-11-411-*t*.)

J. R. KLINE,
Associate Secretary

THE NOVEMBER MEETING IN LAWRENCE

The three hundred thirty-sixth meeting of the Society was held at the University of Kansas on Friday and Saturday, November 27–28, 1936. About one hundred persons attended among whom were the following fifty-two members of the Society:

V. W. Adkisson, R. W. Babcock, Wealthy Babcock, F. L. Black, L. M. Blumenthal, W. C. Brenke, C. C. Camp, R. D. Daugherty, G. M. Ewing, L. R. Ford, M. G. Gaba, G. D. Gore, Cornelius Gouwens, W. A. Harshbarger, J. J. L. Hinrichsen, D. L. Holl, H. M. Hosford, A. S. Householder, Emma Hyde, M. H. Ingraham, H. E. Jordan, A. J. Kempner, W. S. Kimball, W. A. Luby, Anna Marm, Mary N. Arnoldy, U. G. Mitchell, T. A. Mossman, G. C. Munro, J. L. Nagle, G. D. Nichols, O. J. Peterson, T. A. Pierce, P. S. Pretz, B. L. Remick, D. H. Richert, P. R. Rider, H. L. Rietz, P. G. Robinson, W. H. Roever, L. L. Runge, D. T. Sigley, E. R. Smith, G. W. Smith, R. G. Smith, J. Y. Stephens, E. B. Stouffer, W. T. Stratton, C. B. Tucker, W. G. Warnock, M. S. Webster, J. J. Wheeler.

In the history of the Society no other Thanksgiving meeting in the southwest has been attended by as many members, and, as far as is known, no such meeting has had as large a total attendance.

Friday afternoon two addresses were given. Professor H. L. Rietz of the University of Iowa spoke on *Some topics in sampling theory* and Professor Constantin Carathéodory of the University of Munich on *Bounded analytic functions*.

A dinner was held at the University Club Friday evening. Eighty mathematicians and their guests attended. The toastmaster, Dean E. B. Stouffer of the University of Kansas, called first upon Professor Constantin Carathéodory of the University of Munich who is Carl Schurz Memorial Professor at the University of Wisconsin for the first semester of this academic year. Professor Carathéodory discussed the social aspects of life among the mathematicians in Göttingen in the first quarter of this century. Professor U. G. Mitchell of the University of Kansas gave a sketch of the life of Professor C. H. Ashton of the University of Kansas who recently died. Professor M. H. Ingraham of the University of Wisconsin then gave an outline of the activities of the American Mathematical Society.

At the session Saturday morning a lecture on *Fractions* was delivered by Professor L. R. Ford of Rice Institute.

Friday afternoon Dean E. B. Stouffer and Professor A. J. Kempner presided. Professors H. L. Rietz, W. H. Roever, and W. C. Brenke presided over the Saturday morning session.

The titles of papers read at the meeting follow. Those whose abstract numbers are followed by *t* were read by title. Papers numbered 1 to 5 were given at the Friday afternoon session and those numbered 6 to 20 at the session Saturday morning.

Paper number 9 was read by Dean E. B. Stouffer and paper number 11 by Professor D. L. Holl. Mr. P. O. Bell was introduced by Dean E. B. Stouffer, Professor Gordon Fuller by Professor M. H. Ingraham, Mr. E. W. Emery by Dean E. B. Stouffer, Dr. V. P. Jensen by Professor D. L. Holl, and Dr. D. Duffin by Dr. D. G. Bourgin.

1. *A characteristic property of the directrix of Wilczynski*, by Mr. P. O. Bell. (Abstract 42–11–433.)

2. *Transformations of a surface bearing a family of asymptotic curves*, by Professor G. D. Gore. (Abstract 42–11–434.)

3. *Orders for which there exist exactly six groups*, by Dr. D. T. Sigley. (Abstract 42–11–432.)

4. *The dependence of a focal point upon curvature in the calculus of variations*, by Professor A. S. Householder. (Abstract 42–11–442.)

5. *Sufficient conditions for a non-regular problem in the calculus of variations*, by Dr. G. M. Ewing. (Abstract 42–11–436.)

6. *Transformation of a system of partial differential equations into tensor form*, by Professor Gordon Fuller. (Abstract 42–11–445.)

7. *Expansions involving differential equations in which the coefficient of the parameter changes sign*, by Professor C. C. Camp. (Abstract 42–11–446.)

8. *Derivatives of line integrals*, by Professor W. S. Kimball. (Abstract 42–11–440.)

9. *Note on the characterization of Sannia's tetrahedron O*, by Dean E. B. Stouffer and Mr. E. W. Emery. (Abstract 42–11–437.)

10. *New metric foundations of hyperbolic geometry*, by Professor L. M. Blumenthal. (Abstract 42–11–393.)

11. *An application of non-analytic functions in plane stress problems*, by Dr. V. P. Jensen and Professor D. L. Holl. (Abstract 42–11–438.)

12. *The deviating force on a rapidly moving aeroplane due to the earth's rotation*, by Professor W. H. Roever. (Abstract 42–11–448.)

13. *Triple systems as ruled quadrics*, by Professor W. G. Warnock. (Abstract 42–11–435.)

14. *Orthogonal polynomials with orthogonal derivatives*, by Dr. M. S. Webster. (Abstract 42–11–439.)

15. *A note on horn angles and abstract metrics*, by Professor L. M. Blumenthal. (Abstract 42–11–392–*t*.)

16. *Counterpoints and associated cubics*, by Dr. J. M. Feld. (Abstract 42–11–431–*t*.)

17. *On symmetric ruled surfaces*, by Professor Arnold Emch. (Abstract 42–11–441–*t*.)

18. *Projective analogues of descriptive systems of axioms in the foundations of geometry*, by Dr. A. R. Schweitzer. (Abstract 42–11–443–*t*.)

19. *Some theorems concerning non-associative rings*, by Professor N. H. McCoy. (Abstract 42–11–444–*t*.)

20. *A method for certain two-point boundary value problems*, by Dr. D. G. Bourgin and Dr. R. Duffin. (Abstract 42–11–447–*t*.)

M. H. INGRAHAM,
Associate Secretary

THE NOVEMBER MEETING IN LOS ANGELES

The three hundred thirty-seventh meeting of the American Mathematical Society was held at the University of California at Los Angeles on Saturday, November 28, 1936. The meeting was called to order at 9:30 A.M., and was presided over by Professors E. R. Hedrick and G. C. Evans. About fifty persons were in attendance, including the following twenty-six members of the Society:

Clifford Bell, E. T. Bell, Myrtie Collier, P. H. Daus, G. C. Evans, H. E. Glazier, H. J. Hamilton, E. R. Hedrick, M. R. Hestenes, G. H. Hunt, C. G. Jaeger, Glenn James, Hans Lewy, C. F. Luther, W. E. Mason, A. D. Michal, T. M. Putnam, A. A. Shaw, G. E. F. Sherwood, D. V. Steed, A. E. Taylor, S. E. Urner, Morgan Ward, W. M. Whyburn, E. R. Worthington, Max Zorn.

Luncheon for members and their guests was served at Kerckhoff Hall. Titles of papers read at the meeting follow. Those whose abstract numbers are followed by the letter *t* were read by title. Messrs. Hyers, Paxson, and Elconin were introduced by Professor Michal.

1. *A priori limitations for solutions of Monge-Ampère equations.* II, by Dr. Hans Lewy. (Abstract 42-11-449.)

2. *A direct sufficiency proof for the problem of Bolza in the calculus of variations*, by Dr. M. R. Hestenes. (Abstract 42-11-450.)

3. *On Measures and Weights by Epiphanius*, by Professor A. A. Shaw. (Abstract 42-11-451.)

4. *The ternary cubic representing norm and its hessian*, by Professor P. H. Daus. (Abstract 42-11-452.)

5. *Note on functional equations of a cyclic type*, by Professor W. M. Whyburn. (Abstract 42-11-453.)

6. *The ranks of apparition of primes in a linear divisibility sequence*, by Professor Morgan Ward. (Abstract 42-11-454.)

7. *A higher upper limit to the parameters in Fermat's equation* $x_1+x_2^n=x_3^n$, by Professor Glenn James. (Abstract 42-11-455.)

8. *Differential invariants in an abstract differential geometry*, by Professor A. D. Michal and Mr. D. H. Hyers. (Abstract 42-11-456.)

9. *Existence theorems for ordinary differential equations in topological spaces* (preliminary report), by Mr. E. W. Paxson. (Abstract 42-11-457.)

10. *Analytic functionals defined in Hilbert space* (preliminary report), by Dr. A. E. Taylor. (Abstract 42-11-458.)

11. *A remark on real division algebras*, by Professor Max Zorn. (Abstract 42-11-459.)

12. *Certain ternary cubic arithmetical forms*, by Professor E. T. Bell. (Abstract 42-11-460.)

13. *Composite operators*, by Mr. Victor Elconin. (Abstract 42-11-461.)

14. *On differential geometry in the large*, by Dr. Hans Lewy. (Abstract 42-11-463-*t*.)

15. *Generalized integrals and differential equations*, by Dr. Hans Lewy. (Abstract 42-11-462.)

16. *Waring's problem for almost nth powers* (preliminary report), by Mr. Alvin Sugar. (Abstract 42-11-464-*t*.)

17. *On bilinear formulas and Laplace transforms*, by Professor Einar Hille. (Abstract 42-11-465-*t*.)

18. *On a problem in additive number theory* (preliminary report), by Professor R. D. James. (Abstract 42-11-466-*t*.)

19. *A geometric interpretation of the convergents for a ternary continued fraction* (preliminary report), by Professor P. H. Daus. (Abstract 42-11-467-*t*.)

20. *On certain configurations of points in space and linear systems of surfaces with these as base points*, by Professor Arnold Emch. (Abstract 42-11-468-*t*.)

21. *Lebesgue integrals for functions out of a quasi-convex linear topological space* (preliminary report), by Mr. D. H. Hyers. (Abstract 42-11-469-*t*.)

22. *Differential geometry of an abstract topological group with Banach coordinates* (preliminary report), by Professor A. D. Michal. (Abstract 42-11-470-*t*.)

23. *Gateaux differential equations in Banach spaces*, by Mr. D. H. Hyers. (Abstract 42–11–471-*t*.)

24. *The Wronskian of the functions* $x^{k-1}e^{a_i x}$, by Mr. Victor Elconin. (Abstract 42-11-472-*t*.)

25. *Linear paraphrasing*, by Mr. Victor Elconin. (Abstract 42-11-473-*t*.)

26. *Euler's integrability condition*, by Mr. Victor Elconin. (Abstract 42-11-474-*t*.)

T. M. PUTNAM,
Associate Secretary

SHORTER NOTICES

Storia delle Matematiche. By Gino Loria. Volume 3. Dell' Alba del Secolo XVIII al Tramonto del Secolo XIX. Torino, 1933. 607 pp.

The third volume of Loria's *History of Mathematics* is a worthy completion to a most interesting and valuable work. The first two volumes have been reviewed in this Bulletin.* They brought the account of the main lines of mathematical development down to the end of the 17th century. The present final volume discusses the 18th and 19th centuries. The same qualities of vividness and clarity that distinguished the other volumes are maintained here.

Chapters 31 and 32 continue the account of the development of calculus, begun in Volume 2, and carry it on to the time of Euler. Here the arrangement is such as to bring out the progress made in the several countries, Switzerland, England, Italy, and France. In the first the chief names are Jean Bernoulli and his successors in that famous mathematical family. As to England, an adequate summary is given of the debate regarding the validity of Newton's *Fluxions* which has been still more completely discussed for English readers by Cajori in his *History of the Conceptions of Limits and Fluxions . . . from Newton to Woodhouse.*† The chief writers treated from Italy and France are Grandi, Rolle, and Clairaut.

Chapter 33 describes the early stages of the development of the theory of probability, with a brief but satisfactory account of the *Ars Conjectandi* of Jacques Bernoulli.

Chapter 34 is devoted to an account of the life and some of the most significant works of Euler; Chapter 35 to his contemporaries. As more than 30 pages are given over to the most important of Euler's discoveries in the various fields, it may be granted that even this extraordinarily fertile genius has received adequate attention. The chapter on Euler's contemporaries discusses the work of Saccheri, d'Alembert, Condorcet, Lambert, Cramer, and others.

Chapters 36 and 37 discuss Lagrange and his contemporaries. Laplace receives a very "stepmotherly" treatment with a page and a half, which is insufficiently explained by the statement that his work was largely in applied mathematics—but this is surely not quite consistent with the final judgment (page 267) "It has been said, and not mistakenly, that the theory of probability owes more to Laplace than to any other man." The great *Théorie Analytique* is only mentioned, not described or even summarized.

Chapter 38 gives an admirable account of the beginnings of "modern geometry" such as we should expect from Professor Loria, who has for so many years given proofs of his mastery of this field both in mathematics and in the history of mathematics. The greatest names in this chapter are Monge, Carnot, Gergonne, and Poncelet. This chapter is continued by Chapters 42 and 43, which show how Chasles, Steiner, von Staudt, and Cremona added to the structure which had been so adequately founded by Monge and Poncelet; and

* Vol. 36 (1930), pp. 336–337; and vol. 38 (1932), pp. 787–788.

† Reviewed in this Bulletin, vol. 27 (1921), pp. 468–470.

how Pluecker, Cayley, Klein, and others, using analytic methods freely, still further extended this palace of pure reason. Non-euclidean geometry and n-dimensional geometry, quaternions, and Lie's and Grassmann's theories, are shown in their setting.

Chapters 39, 40, and 41 are inserted between the first and second parts of the discussion of modern geometry (which does not imply that this order is necessarily to be criticized, as *some* regard must surely be paid to chronology). They take up respectively German mathematics, culminating with Gauss; the beginnings of modern analysis under Bolzano, Cauchy, Abel, and Jacobi, and mathematical physics, with Fourier, Ampère, Poisson (why not Laplace, who seems to be missing from this chapter?), Green, Stokes, Maxwell, Kelvin, and Helmholtz, to mention only the most outstanding names.

Chapter 44 is devoted to the later 19th century's contributions to analysis, and probably comes too near our own times to enable one to make a very just estimate of the relative value of extensive additions and improvements that have been introduced, and which Professor Loria indicates in about fifty pages. The final chapter, Chapter 45, deals with the historians of mathematics, in fifteen pages.

The three-volume history of Loria's must be regarded as not only the most recent but one of the most valuable of the general histories of mathematics; and it would seem eminently desirable that an English version should be published and made available to every student of our science who has any interest in the study of its growth and development.

R. B. McClenon

Les Corps Algébriques et La Théorie des Idéaux. By O. Ore. (Mémorial des Sciences Mathématiques, Fascicule LXIV.) Paris, Gauthier-Villars, 1934. 71 pp.

This book contains a clear and concise development of the fundamentals of the theory of algebraic numbers and the theory of ideals in algebraic fields. The author begins with an account of rational approximation and criteria for algebraic numbers. The second chapter contains an exposition of the fundamental properties of algebraic fields, their integral bases and discriminants. Minkowski's theorem on linear forms is stated without proof. Chapter 3 is an account of the theory of ideals. The author gives a simple proof for the theorem of unique prime ideal decomposition based on ideas of Krull and van der Waerden. The treatment of residue classes and congruences for ideal moduli is complete and concise in Chapter 4. In Chapter 5 the author gives a new proof of the Dirichlet theorem on units based on a neat generalization of the Kronecker theorem identifying as roots of unity all integers whose conjugates lie sufficiently near the unit circle.

Although the book does not pretend to be exhaustive and subjects such as the relation between the defining equation and the arithmetic of the field or the ideal structure of the discriminant divisors are not included, the author has been very successful in making clear many outstanding problems in the subject. An extensive bibliography is appended. The book will be extremely valuable for beginning students.

H. T. Engstrom

Höhere Mathematik. Teil III. *Raumkurven und Flächen. Linienintegrale und mehrfache Integrale. Differentialgleichungen nebst Anwendungen.* By Rudolf Roth. Leipzig, Teubner, 1935. ix+238 pp.

Teil IV, Heft 3. *Aufgabensammlung: Integralrechnung.* Leipzig, Teubner, 1936. 49 pp.

A clear, concise, and careful presentation of topics of the grade of advanced calculus is contained in the first volume under review. The three subjects treated include the fundamental results of differential geometry of the first order with respect to space curves and surfaces, vector methods being used wherever possible; a treatment of line and multiple integrals including the theorems of Stokes, Gauss, and Green; the principal elementary considerations of ordinary differential equations, including indications of approximate and graphic methods of solutions, and a few remarks on partial differential equations, limited mainly to the fundamental equations of mathematical physics. The volume is replete with illustrations from the applications. The author maintains a high standard of rigor in his presentation, including an existence theorem for multiple integrals of continuous functions and for a system of differential equations. There are many mathematical side glances, to add interest to the presentation, for example, a mention of the Möbius strip, Schwarz's proof that the area of a surface is not always the least upper bound of the areas of inscribed polygons, a discussion of the n-dimensional volume of an n-dimensional sphere, a treatment of the problem of pursuit, and so on. The author has produced an excellent treatise in small space, containing more than the essentials of the subjects considered.

The second volume under review is a collection of examples to cover the integral calculus part of the treatise. The plan of the collection follows the form of the previous parts, exercises being succeeded by their solutions, and many exercises already found in the text being included. The collection is suggestive in giving a number of applications not usually found in texts on the calculus in this country. One defect is that frequently other and better solutions of the problems can be found than those given.

T. H. HILDEBRANDT

Sur les Classes d'Idéaux dans les Corps Quadratiques. By S. Iyanaga. (Actualités Scientifiques et Industrielles, No. 197.) Paris, Hermann, 1935. 13 pp.

This pamphlet, one of the series published in memory of Jacques Herbrand, contains a discussion of the following theorem due to L. Rédei: A necessary and sufficient condition for the existence of an ideal class (in the restricted sense) of order 4 in a quadratic field $k(\sqrt{D})$ is that the discriminant of the field may be written as the product of two factors, $D=D_1D_2$, such that all the prime factors of the discriminants of $k(\sqrt{D_1})$ and $k(\sqrt{D_2})$ decompose completely in $k(\sqrt{D_2})$ and $k(\sqrt{D_1})$ respectively. The proof, based on the translation theorem of Hasse for class fields, is given in detail and in simple form. The form of the proof leads to a generalization relating the number of basis elements of order 2^r, $r\geqq 2$, of the class group of $k(\sqrt{D})$ and the number of decompositions of D with the Rédei property. An appendix contains applications to the determination of the sign of the fundamental unit in real quadratic fields.

H. T. ENGSTROM

Les Théorèmes de la Moyenne pour les Polynomes. By J. Favard. (Actualités Scientifiques et Industrielles; Exposés sur la théorie des fonctions, publiés sous la direction de Paul Montel.) Paris, Hermann, 1936. 51 pp.

The study of the location in the complex plane of the roots of a given polynomial is an interesting and important topic, but perhaps even more interesting geometrically is the study of the mutual relations between the roots of two related polynomials; for instance, one polynomial may be the derivative of the other, or the coefficients of the two polynomials may satisfy a bilinear relation.

The literature on these topics was already extensive at the end of the last century, but has increased enormously since 1900 and is still growing rapidly. At the center of the modern theory lies the theorem of Grace (1902): *If two polynomials*

$$a_0 + C_{n,1}a_1x + C_{n,2}a_2x^2 + \cdots + a_nx^n,$$
$$b_0 + C_{n,1}b_1x + C_{n,2}b_2x^2 + \cdots + b_nx^n,$$

are apolar:

$$a_0b_n - C_{n,1}a_1b_{n-1} + C_{n,2}a_2b_{n-2} + \cdots + (-1)^n a_nb_0 = 0,$$

then no circle can separate the roots of one polynomial from the roots of the other.

In the essay under review the author sets forth various modern generalizations to polynomials of the mean value theorem of the differential calculus. In the complex domain an illustration is Grace's application of the theorem already stated: *If a polynomial $p(z)$ of degree n takes equal values in the points $+1$ and -1, then the derivative has at least one root on or within the circle whose center is the origin and radius* $\operatorname{ctn}(\pi/n)$. Other results in this domain of ideas, clearly presented by Favard, are due to Heawood, Kakeya, Biernacki, Szegö, Montel, Alexander, and others.

In the real domain there presents itself a problem related both to Grace's theorem and the mean value theorem: *Characterize the sets of real constants $c_0, c_1, c_2, \cdots, (c_0>0)$, such that every polynomial with real coefficients*

$P(z) \equiv a_0 + a_1z + a_2z^2 + \cdots + a_nz^n$, *where* $a_0c_0 + a_1c_1 + \cdots + a_nc_n = 0$,

has at least one real root. Favard has already given an elegant answer: *It is necessary and sufficient that there exist a solution Ψ to the problem of moments*

$$\int_{-\infty}^{\infty} x^n d\Psi = c_n.$$

In this essay further interesting results and applications follow in a most natural manner from formulas of mechanical quadratures. The present knowledge on this topic is due in successive stages to Pompeiu, Montel, Biernacki, Tchakaloff, Favard; related theorems involving more general functions are due to Schoenberg. Direct extensions to the complex plane have as yet not been made; but certain known results (Carathéodory) for the case of the circle have some significance.

Favard's exposition is clear and precise, stimulating in its suggestion of unsolved problems, valuable for a number of new methods and results.

There is a crying need for further capable efforts to continue the unification of the entire subject of the location of roots!

J. L. WALSH

Integralgleichungen. By G. Hoheisel. Berlin and Leipzig, de Gruyter, 1936. 136 pp.

This little book is volume 1099 of the familiar Sammlung Göschen. As in other numbers of this series, the printer has made the maximum use of available space with the result that a surprising amount of material is to be found in a convenient pocket-size volume.

The author begins with an introduction to the geometry of function spaces with particular reference to functions of integrable square. This is followed by four chapters with the titles: 1. *Linear integral equations with arbitrary kernel*; 2. *The hermitian kernel*; 3. *The orthogonal system of characteristic functions*; 4. *Particular kernels.*

Chapter 1 begins with a brief sketch of the method of Fredholm involving linear systems of algebraic equations. This point of view is introduced merely for historical and pedagogical reasons and is not used in the sequel. The existence and properties of the solving function (lösende kern) are demonstrated and its practical determination is given by means of Neumann's series. Chapter 2 contains the expansion theorems which are important in mathematical physics and other applications. In Chapter 3 is found a brief treatment of the properties of orthogonal systems of characteristic functions. The last chapter discusses the equation of Volterra, symmetrisable kernels, and certain singular kernels.

This volume is a worthy successor to the large number already published in this series.

W. R. LONGLEY

Cycles of Reduced Ideals in Quadratic Fields. By E. L. Ince. (British Association for the Advancement of Science, Mathematical Tables, Volume IV.) London, British Association, 1934. 80 pp.

This volume contains a tabulation of the cycles of reduced ideals in the real quadratic fields $R(\sqrt{m})$ for all square-free integers m less than 2025. The ideals of each cycle are indicated by minimal bases. The table gives for each field the number of genera and the number of ideal classes in each genus where equivalence is used in the wide sense, as well as the fundamental unit of each field. The class to which a cycle belongs is indicated in such a way that the structure of the group of ideal classes is apparent. The generic character of each cycle is also tabulated.

The book contains a detailed account of the generation of the cycles of reduced ideals and of the structure of the tables. The only previous tables in this connection are those of the solution of the equation of Pell $x^2-my^2=1$. These are of course related to the ideals of the cycle in the principal class. The present volume contains the first tabulation of the secondary cycles. The work has been carefully calculated and checked with the existing tables of the Pell equation where possible. The book will be of great assistance to workers in the theory of numbers and the theory of quadratic forms.

H. T. ENGSTROM

NOTES

The editors of the Annals of Mathematics desire to secure the following numbers of that journal: vol. 1, No. 1; vol. 2, Nos. 1, 3; vol. 3, Nos. 2–4; vol. 4, No. 1.

A new mathematical journal entitled the Revue Mathématique de l'Union Interbalkanique is now being published in Athens. The first number appeared in November, 1936.

The Adams Prize at the University of Cambridge is open to the competition of all persons who have at any time been admitted to a degree at the University. The subject proposed for the period 1937–38 is the distributional properties of functions of statistical variables.

The Royal Society of London has awarded a Royal Medal to R. H. Fowler for his work in statistical mechanics and allied departments of modern mathematical physics.

One of the Townsend Harris Prizes of the College of the City of New York has been awarded to Professor Edward Kasner, of Columbia University, for his achievements in the field of mathematics.

Professor Felix de Roy, president of the commission on meteorites, zodiacal light, and related phenomena, and director of the Variable Star section of the British Astronomical Association, was recently honored with the degree of Doctor of Mathematics and Physics by the University of Utrecht in recognition of his astronomical work. This honor was conferred in connection with the celebration of the tercentenary of that university.

Dean G. D. Birkhoff, of Harvard University, has been selected by Pope Pius as a member of the newly formed Pontifical Academy of Sciences composed of seventy world-famed scientists. Members were chosen solely for their outstanding scientific achievement.

Professor Richard Becker, of the Berlin Technical School, has been appointed to a professorship at the University of Göttingen.

The title of emeritus professor of statistics in the University of London has been conferred on Professor A. L. Bowley, on his retirement from the University chair of statistics at the London School of Economics.

Professor S. Chapman, of the Imperial College of Science, has been elected a member of the Kaiserlich Deutsche Akademie der Naturforscher, Halle, in recognition of his researches in terrestrial magnetism.

Professor R. A. Fisher, Galton Professor of Eugenics at University College, London, is a visiting professor at the University of California for the academic year, 1936–37.

Dr. L. W. Nordheim, formerly lecturer in theoretical physics at the University of Göttingen, is visiting professor at Purdue University for this academic year.

Mr. Daniel Pedoe, last year at the Institute for Advanced Study, has been appointed Demonstrator at University College, Southampton, England.

Dr. W. G. Penney, Stokes student at Pembroke College, Cambridge, has been appointed reader in mathematics in the Imperial College, Royal College of Science.

Professor Erhard Tornier, of the University of Göttingen, has been appointed professor of applied mathematics at the University of Berlin.

At the Oklahoma Agricultural and Mechanical College Associate Professors E. F. Allen and W. V. N. Garretson have been promoted to professorships. Dr. L. S. Stephens has been promoted to an assistant professorship.

Assistant Professor H. E. Arnold, of Wesleyan University, Middletown, Conn., has been promoted to an associate professorship.

Professor Samuel Beatty, of the University of Toronto, has been appointed dean of the faculty of arts at that institution.

Dr. H. C. Carter, of the University of Missouri, has been appointed to a professorship at State Teachers College, Fredericksburg, Va.

Dr. H. S. M. Coxeter, of Washington, D. C., has been appointed to an assistant professorship at the University of Toronto.

Dr. James Fisher has been appointed dean of the faculty of the Michigan College of Mining and Technology, Houghton, Mich. Dr. Fisher, who is head of the department of mathematics and physics, has been associated with the teaching staff since 1896.

Dr. Frances Harshbarger, of Kent State University, has been promoted to an assistant professorship.

Dr. C. F. Luther, of Stanford University, has been appointed to an associate professorship at Willamette University.

The retirement of Professor M. A. MacKenzie, of the University of Toronto, is announced.

Emeritus Professor W. D. Macmillan, of the University of Chicago, has been appointed to a lectureship at Northwestern University for the first semester of this year.

Dr. R. S. Martin, last year at the Institute for Advanced Study, has been appointed an associate at the University of Illinois for the academic year 1936–37.

Dr. R. L. Mennet, professor of mathematics at Tulane University, has been appointed acting president of the university for the academic year.

Associate Professor R. M. Peters, of Judson College, Alabama, has been appointed to a professorship in mathematics and physics at Lake Erie College.

Assistant Professor G. E. Raynor, of Lehigh University, has been promoted to an associate professorship.

Dr. Robin Robinson, of Dartmouth College, has been promoted to an assistant professorship.

Professor T. R. Running, of the University of Michigan, is retiring from active teaching in February, 1937.

Assistant Professor N. E. Rutt, of Northwestern University, has been appointed to an associate professorship at Louisiana State University.

Assistant Professor C. A. Shook, of Lehigh University, has been promoted to an associate professorship.

Professor W. G. Simon, of Adelbert College and Western Reserve University, has been elected dean of the faculties of arts and sciences for Adelbert and Mather Colleges and the Graduate School of Western Reserve University.

Dr. A. J. Smith, of the University of Pennsylvania, has been appointed professor of mathematics at Susquehanna College, Selinsgrove, Pa.

The retirement on July 1, 1936, of Professor P. F. Smith, of Yale University, is announced.

The following members of the American Mathematical Society are in residence at the Institute for Advanced Study this academic year on leave of absence from their respective institutions: Professor J. H. Bartlett, of the University of Illinois; Professor S. S. Cairns, of Lehigh University: Professor W. L. Duren, of Tulane University; Dr. P. W. Ketchum, of the University of Illinois; Professor E. J. Moulton, of Northwestern University; Professor F. D. Murnaghan, of Johns Hopkins University; Dr. Walter Prenowitz, of Brooklyn College; Dr. J. F. Randolph, of Cornell University; Professor M. H. Stone, of Harvard University; and Professor J. M. Thomas, of Duke University.

The following mathematicians from foreign universities are also in residence at the Institute: Professor Péi-Yüan Chou, of the National Tsing Hua University, Peiping, China; Professor G. H. Hardy, of Trinity College, England; Dr. Witold Hurewicz, of the University of Amsterdam, Holland; Dr. Leopold Infeld, of the University of Lwow, Poland; Professor Tsai-Han Kiang, of the National University of Peking, China; Professor Tullio Levi-Civita, of the University of Rome.

The following appointments to instructorships are announced: University of Michigan: Dr. S. B. Myers, Mr. M. E. Shanks; Ohio University: Mr. Carl Denbow; Pennsylvania State College: Dr. Aline H. Frink; Pennsylvania State College, Hazelton Extension Center: Mr. Coleman Herpel; Tulane University: Dr. Charles Hopkins.

Mr. C. E. Haselfoot, fellow of Hertford College, Oxford, and formerly

lecturer in mathematics in Wadham College, died on October 28, 1936, at the age of eighty-two years.

The death of Mr. C. M. Biscay, of the Western and Southern Life Insurance Company, is announced. He had been a nominee of the company since 1927.

The death of Mrs. M. H. Carus, of La Salle, Ill., is reported.

The death of Professor N. A. Patillo, of Randolph Macon College, is reported.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

1. Professor C. R. Adams and Mr. A. P. Morse: *On the space* (BV).

The authors consider the space (BV) of functions $x(t)$ of bounded variation on $0 \leqq t \leqq 1$, metrized thus: $(x, y) = \int |x(t) - y(t)|\, dt + |T(x) - T(y)|$ (see Adams, Transactions of this Society, November, 1936). The following semi-continuity property of the functional $T(x)$ [also enjoyed by $L(x)$, the Peano length of $x(t)$] is basic: $x_n(t) \to x(t)$ on a dense set, with $x(t)$ continuous at the end-points and having an external saltus at no interior point, implies $\underline{\lim}\ T(x_n) \geqq T(x)$. Several questions naturally raised by the paper cited may now be answered, and it can be shown that the set (DBV) = (the discontinuous elements of (BV)) is the sum of non-dense closed sets in (BV); whence (CBV) = (the continuous functions in (BV)) is a residual set in (BV). Likewise $(CBV - CS)$, where (CS) = (the continuous singular functions), is the sum of non-dense closed sets in (CBV); and (CS) is a residual set in (CBV). Schauder's base for (C) is a base also for (CBV); and Haar's system of orthogonal functions, augmented by the characteristic function of the set $\{1\}$, provides a base for (BV). (Received November 30, 1936.)

2. Professor V. W. Adkisson and Dr. Saunders MacLane: *On planar graphs whose homeomorphisms can all be extended.*

This paper considers the non-separable planar graphs such that, regardless of how the graph is mapped on the sphere, all the homeomorphisms of the graph into itself can be extended to the sphere. The idea of proper components is utilized. A graph G is split into two proper components, H_1 and H_2, if $G = H_1 + H_2$, if neither H_1 nor H_2 is a single edge, and if H_1 and H_2 have in common only two vertices (the "end" vertices). A proper component is minimal if no split of the graph gives a component which is a proper subset of H_1. If, regardless of how G is mapped on the sphere, all its homeomorphisms are extendible then either (1) G cannot be split into proper components, (2) G has no non-trivial homeomorphisms, (3) G contains only two minimal components of a specified type, (4) G has just one non-trivial homeomorphism of order two, and this homeomorphism merely exchanges the end vertices of each minimal component (the vertices the component has in common with the rest of G), with no rotation of the element with respect to these vertices. Conversely, if

any one of the above conditions is satisfied, then all homeomorphisms of G into itself are extendible regardless of how G is mapped. (Received November 27, 1936.)

3. Professor W. L. Ayres: *On transformations having periodic properties.*

Using the Whyburn cyclic element theory a study is made of continuous transformations of compact locally connected spaces into themselves which are periodic, point-wise periodic, almost periodic, or point-wise almost periodic. While these four types of transformations are quite different, it is found that they have identical properties with regard to the cyclic elements. Among the results are the following: the set of cyclic elements which are invariant in the large is non-vacuous, closed, and connected. If A and B are cyclic elements such that $f(A)=B$, then the cyclic chain from A to B contains one and only one invariant cyclic element. (Received November 30, 1936.)

4. Mr. Garrett Birkhoff: *Analytical groups.*

A class of abstractly defined "analytical groups" is defined, embracing Lie groups and the infinite continuous groups studied by Delsarte (but not those studied by Lie and Cartan). These groups are found to possess many properties of Lie groups, namely, one-one representability by canonical parameters, continuous homeomorphism with an adjoint, existence of a Lie algebra, correspondence between Lie subalgebras and subgroups, invariant Lie subalgebras and invariant subgroups, analytical function of composition under canonical parameters, and so on. Also, it is pointed out that the invariants of Lie groups are all topologico-algebraic. Further, an infinitesimal calculus for analytical groups is developed, which extends the Riemann-Volterra and the Lebesgue-Schlesinger theories for matrices to non-linear groups and to infinite continuous groups. Various results ignored or overlooked in the case of matrices are also given. (Received November 18, 1936.)

5. Mr. Garrett Birkhoff: *On the integral calculus of operators.*

An answer is given to the following question. Let $X(t)=\sum_{k=1}^{r}\rho_k(t)\cdot X^k$ be a variable linear combination of several fixed operators X^k operating over a specified period of time. What single fixed operator Z, operating constantly over the same period, will produce the same net effect? The computation of Z involves only the addition, multiplication, quadrature, and differentiation of the $\rho_k(t)$, and the taking of successive Poisson brackets of the X^k. (Received November 18, 1936.)

6. Professor H. E. Bray: *On the roots of the derivative of a polynomial.*

The author proves the following theorem: If $z_1, z_2, \cdots, z_n$, are the roots of a polynomial $P(z)$, and if ξ is a root of the derivative $P'(z)$ but not of $P(z)$, and if $|z_1|<|z_2|<\cdots<|z_n|$ are held fixed, then $|\xi|$ attains its greatest possible value when and only when $z_1, z_2, \cdots, z_n$, all have the same amplitude. The properties of the cubic curve (in polar coördinates) $r=a\sin(\alpha-\theta/2)/\sin(\theta/2)$ provide the essential facts in the proof. (Received November 30, 1936.)

7. Dr. E. A. Cameron and Professor J. W. Lasley: *On certain loci associated with a plane curve.*

The equations of conics of various orders of contact are obtained in both local and general systems of coordinates. A study is made of properties and relations of these osculants and other connected loci at the point. As the point moves along the curve associated loci are generated by parts of penosculating conics. The equations of a number of such loci have been obtained and their properties established. (Received November 27, 1936.)

8. Dr. R. H. Cameron: *Quotients of almost periodic functions.*

This paper deals with solutions $x(t)$ of the equation $f(t)x(t)\equiv g(t)$, in which $f(t)$ and $g(t)$ are uniformly almost periodic functions and $g(t)$ has no translation transform which vanishes throughout an interval. It is shown that a necessary and sufficient condition that $x(t)$ be uniformly almost periodic is that it be bounded and uniformly continuous. (Received November 23, 1936.)

9. Dr. R. H. Cameron and Dr. W. T. Martin: *Analytic continuation of diagonals and Hadamard compositions of multiple power series.*

The authors consider an analytic function $A(x, y)=\sum a_{mn}x^m y^n$ and its star S, a region which consists of all points (x_0, y_0) such that A is regular for $x=\rho x_0$, $y=\sigma y_0$ $(0\leqq\rho\leqq 1, 0\leqq\sigma\leqq 1)$. They define a *contracted* star S^* which is the region in the z-plane consisting of all points z for which there exists a continuous positive periodic function $r(\theta)$ such that all of the points $(z_n(\theta)e^{i\theta}, e^{-i\theta}/n(\theta))$ are in S. It is shown that the *diagonal function* $A^*(w)=\sum a_{nn}w^n$ *is regular in* S^*. *When* $A(x, y)=B(x)C(y)$ *this result gives the classical Hadamard result.* By iteration of this theorem it is shown that the diagonal function $A^{**}(z, w)=\sum a_{nnnn}z^n w^n$ of a function $A(x, y, u, v)$ is analytic in a star-shaped region characterized in terms of the star of $A(x, y, u, v)$. In particular, if $A(x, y, u, v)=B(x, u)C(y, v)$ then the diagonal function is the two-variable analogue of the Hadamard composition, namely $\sum b_{mn}c_{mn}z^m w^n$. By continued iteration, these results can be carried over to any number of variables. (Received November 27, 1936.)

10. Professor Leonard Carlitz: *A class of polynomials.*

For $\psi(t)$ as defined in the Duke Mathematical Journal (vol. 1 (1935), p. 148) the polynomials $\omega_M(t)$ are defined by $\psi(Mt)=\omega_M(\psi(t))$. In turn polynomials $W_M(t)$ are defined by $\omega_M=\prod W_A$, the product extending over all A dividing M, where M is an arbitrary polynomial in $GF(p^n)$. In this paper ω_M is defined algebraically without the use of the function $\psi(t)$, and various properties of W_M deduced. The polynomials W_M are seen to be analogous to the cyclotomic polynomials. (Received November 18, 1936.)

11. Professor Leonard Carlitz: *A criterion for certain higher congruences.*

The congruences in question are $\sum_{i=0}^{s}(-1)^i[s^s-i]^{p^i}u^{p^i}\equiv M^{p^s}$ (mod P), where M is an arbitrary polynomial in a single indeterminate x with coefficients

in a finite field, and P is an irreducible polynomial of this kind; the coefficients $[s^n-i]$ are certain polynomials in x defined explicitly in the Duke Mathematical Journal (vol. 1 (1935), p. 141). If P is of degree $k>s$, it is shown that the congruence is solvable if and only if $P'M$ (where P' denotes the derivative) is congruent (mod P) to a polynomial of degree $<k-s$. Thus this theorem is a direct generalization of a known theorem (the case $s=1$). (Received November 30, 1936.)

12. Professor Leonard Carlitz: *Some formulas for factorable polynomials in several indeterminates.*

A factorable polynomial in $GF(p^n)$ is defined by the product $\Pi_j(\alpha_{j0}+\alpha_{j1}x_1+\cdots+\alpha_{jk}x_k)$, where the α_{ji} are in some $GF(p^{nm})$ (see Duke Mathematical Journal, vol. 2 (1936), No. 4). In this note some formulas for the case $k=1$ (ordinary polynomials in a single indeterminate) are extended to the case $k>1$. (Received November 18, 1936.)

13. Professor J. M. Clarkson: *Cremona transformations of plane curves expressed by third-order differential equations.*

Given three independent solutions x_1, x_2, x_3 of the equation (1) $x'''+3p_1x''+3p_2x'+p_3x=0$, where x_i are functions of a parameter t and where primes indicate differentiation with respect to t, if these x_i be taken as the homogeneous coordinates of a point in the plane, then as t varies, the x_i trace a curve expressed by (1). An investigation is made of the behavior of the coefficients p_i when a Cremona transformation (2) $y_i=y_i(x_1, x_2, x_3)$ is performed on x_i. (Received November 25, 1936.)

14. Dr. W. S. Claytor: *Peanian continua not imbeddable in a spherical surface.*

In a primitive skew curve K let a and b be vertices which bound one of its edges ab, and let C denote the curve $\overline{K-ab}$. If $C_n(=t(C))$, is any topological image of C, then set $a_n=t(a)$ and $b_n=t(b)$. In 3-space take (1) an infinite sequence of curves $\{C_n\}$ each of which is homeomorphic with C, (2) an infinite sequence of arcs $\{b_na_{n+1}\}$, and (3) a single arc PQ; such that (1) $C_iC_j=(b_ia_{i+1})(b_ja_{j+1})=C_i\cdot PQ=(b_ia_{i+1})\cdot PQ=0$, for $i\geqq 1$ and $i\neq j$; (2) $(b_ia_{i+1})\cdot\sum_{n=1}^{\infty}C_n=(b_ia_{i+1})\cdot(C_i+C_j)=b_i+a_{i+1}$, $(i\geqq 1)$; (3) $\lim_{n=\infty}(C_n+b_na_{n+1})=P$. Then set $\Delta=\sum_{n=1}^{\infty}(C_n+b_na_{n+1})+PQ$. It is readily seen (as was shown by Kuratowski, Fundamenta Mathematicae, vol. 15, p. 272, footnote 2), that Δ is a curve which does *not* have a homeomorphic image in the plane. Furthermore, since there are just *two* primitive skew curves (topologically speaking), it follows that Δ is homeomorphic with one or the other of two curves which are denoted by Δ_1 and Δ_2. The following theorem is proved: a Peanian continuum which is not homeomorphic with a subset of a spherical surface necessarily contains either a primitive skew curve or a topological image of one of the curves Δ_1 and Δ_2. For the definition of a primitive skew curve, see the paper by this author in the Annals of Mathematics (vol. 35 (1934), No. 4). (Received November 28, 1936.)

15. Professor L. W. Cohen: *Transformations on spaces with denumerable basis.*

Conditions that a linear Tx on a Banach space with denumerable basis to another such space be completely continuous and that the matrix of $(I+T)x$ have an absolutely convergent determinant are given. In the case of Tx on the space l_1 with $\|x\|=\sum|x_i|$ to the space l_0 with $\|x\|=\sup|x_i|$, $\lim x_i=0$, the condition for complete continuity becomes $\lim_n \sup_i \sup_{m>n}|a_{mi}|=0$, where $\|a_{mi}\|$ is the matrix for T. In the case of $(I+T)x$ on l_0 to l_0, the conditions become (1) $\lim_m\sum_i|a_{mi}|=0$, (2) $\sum_i \sup_m|a_{mi}|<+\infty$. (Received December 1, 1936.)

16. Dr. E. G. H. Comfort: *On the preservation of Hölder properties of initial conditions in the solutions of wave equations.*

A function is said to have a Hölder property if it satisfies a Hölder condition. Kellogg has shown (Transactions of this Society, vol. 33 (1931), p. 486–510) that, for harmonic functions in a sphere, Hölder properties of the boundary values (or of their derivatives) imply Hölder properties (same order) of the harmonic function itself (or of the corresponding derivatives) in the closed sphere. The same holds true for the particular wave equation $u_{tt}-u_{xx}=0$, that is, Hölder properties of the initial conditions (or of their derivatives) are preserved with the same order in the solution (or in the corresponding derivatives of the solution). This paper shows that such is not the case for the wave equation $u_{tt}-u_{xx}-u_{yy}=0$. If the Hölder properties of the initial conditions (or of their derivatives) have an order $\alpha>1/2$, then the solution (or its corresponding derivatives) will have a Hölder property of order $\alpha-1/2$. Corresponding results are shown to hold for the wave equations $u_{tt}-u_{xx}-u_{yy}-Ku=0$, and $u_{tt}-u_{xx}-u_{yy}=\phi(x, y, t)$. (Received November 20, 1936.)

17. Professor H. B. Curry: *On the use of dots as brackets in logical expressions.*

The author suggests a slight modification of the Peanese convention which enables one to represent conveniently chains of indefinite length, such as the formula $a_1 \supset (a_2 \supset (\cdots \supset (a_n \supset b)\cdots))$ (even when the a_i are themselves complex), without doing violence to our ordinary algebraic usage. (Received November 21, 1936.)

18. Professor E. L. Dodd: *Some internal and external means arising from the location of frequency distributions.*

The problem of location for frequency functions as introduced by R. A. Fisher leads to generalized means or substitutive means which may be internal or external. A substitutive mean of observations $x_1, x_2, \cdots, x_n$, relative to a function F, is a solution M of $F(x_1, x_2, \cdots, x_n)=F(M, M, \cdots, M)$, as defined by O. Chisini. The external means occur rather naturally with bimodal distributions. Under certain simple conditions the Fisher likelihood process leads to exterior means with maximum likelihood and to interior means with minimum likelihood. For the problems here considered, likelihood is equivalent to the *a posteriori* probability, with the *a priori* probability taken as constant.

With this interpretation, the problem is that of finding the "most probable" location for frequency distributions. (Received November 20, 1936.)

19. Dr. F. G. Dressel: *A note on Fredholm-Stieltjes integral equations.*

If $K(x, y)$ is absolutely continuous with respect to a monotone function $g(y)$ for all x, then the paper shows that $f(x)=m(x)+\lambda\int_a^b f(y)dK(x, y)$ is equivalent to an ordinary Fredholm equation. (Received November 24, 1936.)

20. Dr. D. M. Dribin (National Research Fellow): *Quartic fields with the symmetric group.*

In the present paper the possible types of groups of decomposition, inertia, and ramification that are allowed by the normal extension of a quartic field over the rational numbers having the symmetric group are studied, and those types which may exist are determined. These normal fields N are constructed by forming the direct product of relatively quadratic fields Λ_1 and Λ_2 over a normal sextic field B and insuring that the conductors of Λ_1 and Λ_2 be divisible by the proper powers of certain prime ideals in B. This investigation is closely connected with a paper of Hasse on non-normal cubic fields; problems kindred to that under consideration have been studied by Porusch (metabelian fields of quite special type) and by Rosenblüth (the "quaternionic" field). (Received November 30, 1936.)

21. Dr. Nelson Dunford: *Integration of vector-valued functions.* Preliminary report.

A function $f(P)$ with values in a Banach space X is said to be integrable and belong to the class S_p in case $Tf(P)$ is in L_p for every T in $\bar{X}$. For such functions $\int_e Tf(P)dm$ is continuous in T, and thus defines a point in $\bar{\bar{X}}$. This point is taken as the value of $\int_e f(P)dm$. A similar definition holds for the integral of a numerical function with respect to an abstract-valued measure function. The present paper discusses the properties and uses of the above integrals. (Received November 30, 1936.)

22. Dr. Nelson Dunford: *Linear transformations of sequences.*

A necessary and sufficient condition for a function $y=Tx$ to be linear and continuous on l to $l_p(1\leq p<\infty)$ is that it can be given in the form $\eta_i=\sum_{j=1}^{\infty}a_{ij}\xi_j$ where $\sup_i\{\sum_{j=1}^{\infty}|a_{ij}|^p\}^{1/p}<\infty$. This last constant is the evaluation of $|T|$. For Banach spaces with a base $\{\phi_i\}$ a necessary and sufficient condition for the compactness of a set E is $x=\sum_{i=1}^{\infty}a_i\phi_i$ uniformly with respect to x in E. This yields as necessary and sufficient conditions for the complete continuity of $y=Tx$ on l to l_p the above condition as well as the condition $\lim_{n\to\infty}\sup_i\{\sum_{j=n}^{\infty}|a_{ij}|^p\}^{1/p}=0$. Sufficient conditions for complete continuity of $y=Tx$ on l_p to l_q are also given. (Received November 30, 1936.)

23. Professor L. A. Dye: *A transformation associated with the trisecants of a rational twisted quintic curve.*

This transformation is generated by the use of a (1, 1) correspondence between a pencil of ruled cubic surfaces $|F_3|$ and the trisecants of a rational

twisted curve C_5. The trisecants of C_5 are the simple directrices of $|F_3|$ whose base is the C_5 together with a double line which is the quadrisecant of C_5. The most interesting feature of the transformation is the existence of two ruled surfaces whose generators are fundamental lines of the second species of the transformation. One of these surfaces is the ruled surface of trisecants of C_5 and it is not a principal surface of the transformation. The other surface, which is a principal surface, is generated by the tangent lines to C_5 at the points where it cuts the trisecants which are simple directrices of $|F_3|$. (Received November 17, 1936.)

24. Dr. M. M. Flood: *Column normal matric polynomials.*

The principal questions answered in this paper are: (i) What is the greatest lower bound for $d[PX]$ when P is a given matric polynomial, X is a matric polynomial of prescribed rank, and $d[PX]$ is the degree of the product PX? (ii) How can the degree invariants of the column vector space of $C_r[P]$ be expressed in terms of the degree invariants of the column vector space of P, where P is a given matric polynomial and $C_r[P]$ is its rth compound? (iii) How can the problem of finding the matric roots of a matric polynomial P be reduced to the consideration of regular matric polynomials? (Received November 25, 1936.)

25. Dr. H. H. Goldstine: *The calculus of variations in general analysis.* I.

Making use of E. H. Moore's principle of generalization by abstraction, the author was led to attempt to find the general theory underlying both the calculus of variations and the theory of the minima of functions of several variables. In this paper the author has made a first step in this direction by considering the problem of the minima of functions defined on regions of the space of modular functions. The theory developed here includes as special cases the simple fixed end-point problem of the calculus of variations in which the admissible arcs have Lebesgue square integrable derivatives almost everywhere, the theory of the minima of functions of a finite number of variables, and a similar theory for functions of Hilbert sequences. By combining the method of his thesis with that of the second form of general analysis, the author obtains analogs of the multiplier rule, the Clebsch condition, and the Jacobi-Mayer condition; moreover when the end condition is suitably restricted, sufficient conditions are obtained by expansion methods. The analog of the Jacobi condition is especially interesting because, in the case of the calculus of variations, the author has shown it is a stronger condition than the one ordinarily stated. (Received November 23, 1936.)

26. Mr. C. H. Graves: *On certain surfaces in five-dimensional space.*

In this paper a study is made of certain surfaces in five-dimensional space sustaining families of tritangent curves. A tritangent curve is defined as a curve having the property that the ambient space of three consecutive tangent planes of the surface along the curve is an S_4. After studying the differential geometry of surfaces in S_5, the surface of Veronese is defined and a new characteristic

property given: If a surface of S_5 with $S(2, 0) = S_5$ possesses for each of its points three distinct tritangent curves which are also quasi-asymptotics $\gamma_{1,2}$ it is necessarily a surface of Veronese. The conditions on the coefficients in the system of differential equations defining a surface which are necessary for the surface to sustain tritangent curves are developed in a general manner. The particular case of a surface sustaining five tritangent curves is developed in more detail. The problem of the existence of surfaces other than the surface of Veronese on which there exist families of tritangent curves is considered and three special surfaces are defined, one sustaining three tritangent curves through each point, another sustaining four, and the third surface sustaining five. (Received November 18, 1936.)

27. Professor T. R. Hollcroft: *Contacts of algebraic plane curves.*

Aside from the multiple contacts of conic and cubic, the contacts of algebraic plane curves have been studied but little. Since the proof of a theorem by B. Segre (B. Segre, *Esistenza e dimensione di sistemi continui di curve piane algebriche con dati caratteri*, Rendiconti Accademia dei Lincei, (6), vol. 10 (1929), pp. 36–37, Theorem 2°) relating the number of contacts of two curves to the number of cusps of a third curve whose order is the sum of the orders of the first two, the subject of contacts of two curves has taken on new and greater significance. In this paper limits and upper bounds to the number of contacts of two plane curves are obtained in certain cases. (Received November 30, 1936.)

28. Dr. Charles Hopkins: *Concerning the structure and representations of a certain class of finite rings.*

The author considers a ring O with the following characteristics: the number of elements is finite; every left-hand nilfactor is nilpotent; O contains an element which is not a left-hand nilfactor and also an element which is not a right-hand nilfactor. Then O is a hypercomplex system over a Galois field F (of order p^f) and is the direct sum of two subalgebras F^* and R, where F^* is simply isomorphic with F and R is nilpotent. The elements of O which are congruent modulo R to the principal unit of O constitute under multiplication a group Γ of order p^{nf}, where n is the rank of R over F. The connection between the structure of Γ and the ideal structure of R is investigated; the number of non-equivalent cyclic O-moduli is obtained in terms of certain invariants of R. The group-ring of a finite p-group G over a finite field of characteristic p is a special example of the ring O; accordingly one is able to obtain systematically the known results concerning representations of G by linear transformations over F. (Received November 30, 1936.)

29. Dr. C. C. Hurd: *Asymptotic theory of linear differential equations singular in a parameter and in the variable of differentiation.*

This paper considers linear differential equations whose coefficients are series in negative integral powers of a complex parameter λ and the complex

variable x, with possibly a finite number of positive powers of λ and x present. Under the hypothesis that no root of multiplicity greater than one of the characteristic equation satisfies a certain "auxiliary equation," the existence of a full set of formal solutions is demonstrated. The method of iterations, developed by Trjitzinsky for differential equations, is used to establish the asymptotic character of the solutions. (Received November 23, 1936.)

30. Mr. L. P. Hutchinson: *The Lagrange multiplier theorem for normed vector space.*

On XY, the composite of two linear normed spaces, $f(x, y)$ is of class C' to the space of real numbers and $F(x, y)$ is of class C' to Y. Subject to $F(x, y)=0$, $df=0$ at (x_0, y_0). Then at this point there is a linear functional L such that $LdF=df$. The basis of the proof lies in the Hildebrandt-Graves implicit function theorem and the transformation $L_x=L_yT$ on the spaces of linear functionals on X, Y associated with the transformation $Tx=y$. (Received December 1, 1936.)

31. Professor Dunham Jackson: *Orthogonal polynomials in three variables.*

The extension of the theory of orthogonal polynomials from two variables to three naturally proceeds to a considerable extent automatically. Nevertheless the increase in the number of dimensions gives rise to some new questions of interest, which are discussed in the present paper. As possibilities calling for separate consideration, the domain of orthogonality may be a three-dimensional region, or an algebraic surface, or a non-algebraic surface, or an algebraic curve, or a non-algebraic curve on an algebraic surface, or a curve which does not lie on an algebraic surface. (Received November 30, 1936.)

32. Professor Dunham Jackson: *Polynomial approximation on a curve of the fourth degree.*

In another paper (abstract 42-9-335) the writer has discussed the formal properties of polynomials in two real variables which are orthogonal on a curve in the plane of the variables. In the simplest cases, when the curve is a line segment or a circle, the resulting series developments reduce essentially to Legendre and Fourier series respectively, so that in those cases new problems of convergence do not arise. This paper is concerned with questions of the convergence of polynomial approximation on a particular curve of the fourth degree, for which the results are new in substance as well as in form. (Received November 30, 1936.)

33. Professor Dunham Jackson: *Problems of closest approximation on a two-dimensional region.*

In previous articles (this Bulletin, vol. 39 (1933), pp. 889–906; Transactions of this Society, vol. 40 (1936), pp. 225–251) theorems have been presented specifying upper bounds for the magnitude of polynomials and trigonometric sums normalized with respect to a given weight function, and corresponding inequalities relating to the approximate representation of functions of a single

real variable. Results of similar character are now derived for regions of the (xy) plane. While the extension is immediate in some of its aspects, it also introduces new considerations which give the problem in two variables a degree of interest for its own sake. (Received November 30, 1936.)

34. Professor Fritz John: *Polar correspondence with respect to a convex region.*

Let R be a convex region in n-dimensional projective space. A 1-1 correspondence between the points of R and the hyperplanes outside R will be denoted as a polar correspondence, P. C. A P. C. is called positive if a point and its polar plane are not separated by any other point and its polar plane. It is proved that a positive P. C. is always continuous. A P. C. is called symmetric if in the neighborhood of any point it can be approximated by an ordinary P. C. with respect to a quadric. Every positive mass distribution on R gives rise to a positive symmetric P. C. by choosing as pole of hyperplane p the center of mass P of the distribution with p as plane at infinity. The P. C. thus generated is in the neighborhood of P approximately given by the P. C. with respect to Legendre's ellipsoid of inertia of the mass distribution. (Received November 27, 1936.)

35. Professor Edward Kasner: *General trihorn geometry.*

The configuration formed by three curves starting from a common point in a common direction is a fundamental object in conformal geometry and is termed a *trihorn*. The three horn angles in the trihorn have measures M_{12}, M_{22}, M_{21} as defined in the Proceedings of the National Academy, May, 1936. Any two of the curves determine a linear pencil of curves (wide open position, Schwarzian symmetry and conformal bi-section). Any two linear pencils have a new unique conformal invariant which is defined as the "angle" of the two pencils. Thus we have three new angles α_1, α_2, α_3. The angles are subject to an identity—their product is unity. The three measures, however, are subject only to an inequality relation. The complete geometry of trihorns, which may be called *trihornometry*, studies the interrelations, equalities and inequalities, between the six invariants. In the study of this geometry there is introduced an auxiliary plane with cartesian coordinates $x=\gamma$, $y=d\gamma/ds$, where γ denotes curvature and s arc length. The induced group in this plane is $X=mx+h$, $Y=m^2y+k$. The metric in this plane is defined by the calculus of variation problem $\int dx^2/dy=\int(1/y')dx=$minimum. The medians of a triangle are concurrent so the centroid exists; but the altitudes of a triangle are not concurrent so the orthocenter does not exist. Many other theorems in triangle geometry are extended to conformal theory. (Received November 27, 1936.)

36. Professor Edward Kasner: *Special trihorns.*

A trihorn has three measures (sides) and three angles. It is shown that neither equilateral nor equiangular trihorns exist. If two sides are equal it does not follow that the opposite angles are equal—in fact this never can happen. Isosceles right triangles exist of two distinct types, one with angles 2, 3/2, 1/3, the other with angles -2, 1/2, -1. Triangles can exist with two right angles. Perpendicularity (transversality) is defined by the angle 1/2 or 2. If

two lines are parallel the perpendicular distance from one to the other and the anti-perpendicular distance are in the ratio $-8:1$. The minimum distance is defined by the perpendicular. (Received November 27, 1936.)

37. Dr. P. W. Ketchum: *On the possible behavior of an analytic function at a set of isolated points.*

Several theorems which are related to and in some cases generalizations of the Mittag-Leffler and Weierstrass factor theorems are obtained for an arbitrary isolated set of points $\{a_n\}$. The problem in general terms is to construct, if possible, an analytic function $A(z)$ with certain preassigned properties in the neighborhood of each point a_n. The only permitted singularities of $A(z)$, other than those preassigned, will be at the limit points of $\{a_n\}$. As an example, we construct, for an arbitrary isolated set $\{a_n\}$, a function $A(z)$ with a simple zero at each point a_n and whose level curves $|A(z)|=\eta$ will, for every sufficiently small η, contain separate branches about each point a_n. Applications are made to the problem of simultaneously expanding a set of functions $\{f_n(z)\}$, where $f_n(z)$ is analytic at a_n, in the same series $\Sigma c_m A_m(z)$, the series to converge uniformly to $f_n(z)$ near a_n and the c's to be uniquely determined. (Received November 27, 1936.)

38. Dr. V. V. Latshaw: *On second order adjoint difference systems.*

A definition of second order linear adjoint difference systems is formulated. Explicit conditions for self-adjointness are detailed and two systems fulfilling these conditions are included. (Received November 23, 1936.)

39. Dr. A. N. Lowan: *On the operational determination of Green's functions in the theory of heat conduction.*

A Green's function $G(x, y, z; \xi, \eta, \zeta; t, \tau)$ in the theory of heat conduction for a volume V bounded by a surface S is defined as a solution of the equation (A): $\partial u/\partial t - k\Delta u = 0$ for which (1) $\alpha\partial u/\partial n + \beta u = 0$ over S ($\partial u/\partial n$ signifying differentiation along the normal), and which (2) for $t=\tau$ vanishes everywhere except at some point $P(\xi, \eta, \zeta)$ at which $u=\infty$ in such a manner that (3) the volume integral $\iiint u(x, y, z)dxdydz = 1$. The Green's function G is obtained by the superposition of a "point source" solution v of (A) satisfying (2) and (3) and a solution w of (A) vanishing identically for $t=\tau$ and satisfying the further condition $(\alpha\partial/\partial x + \beta)(v+w)=0$ over S. The object of the present paper is the operational determination of the Green's function. This is obtained by constructing the Laplace transform v^*+w^* of G and then subjecting the now known function w^* to the inverse Laplace transformation. The last task is accomplished with the aid of certain standard theorems of the operational calculus. The method is illustrated for the case of a semi-infinite solid, but may be suitably extended to the case of a sphere, a cylinder, and so on. (Received November 23, 1936.)

40. Dr. Saunders MacLane: *A structural characterization of planar combinatorial graphs.*

This paper develops a criterion that a combinatorial graph be planar, which is stated explicitly in terms of natural structural properties of graphs. A

split of a cyclicly connected graph G is a representation $G=H_1+H_2$, in which the two subgraphs H_1 and H_2 have no arcs and just two vertices p and q in common. The "blocks" of this split are obtained by adding to H_1 and H_2 respectively two new edges with the ends p and q. These two blocks may be split in their turn until we obtain unsplittable blocks or "atoms." These atoms are essentially unique. It is shown that the original graph is planar if and only if its atoms are planar. Furthermore an atom A is planar if and only if the circuits of A which do not separate A can be taken as the domain boundaries of a map of the atom A. This is possible when these circuits contain every edge exactly twice and have no other linear relation modulo 2 (S. MacLane, abstract 40-5-195). A graph is planar if its atoms have this property. (Received November 27, 1936.)

41. Professor J. D. Mancill: *On problems of the calculus of variations for which transversality is equivalent to orthogonality.*

It is well known that if the transversality condition for the minimum of the integral $\int f(x, y, y')dx$ shall be equivalent to orthogonality it is necessary and sufficient that f have the form $g(x, y)(1+y'^2)^{1/2}$. The purpose of this note is to present a rather simple proof of the analogous property for parametric problems in space of any number of dimensions. The same procedure is used to determine the integrand function $F(x, y, x', y')$, that is, in the plane case, for which the tangent of the angle between the direction determined by (x', y') and the corresponding transversal direction at each point is some prescribed function α of the coordinates of the point. The function F in this case is of the form $F(x, y, x', y')=G(x, y)(x'^2+y'^2)^{1/2}\cdot\exp\left[-\text{arc tan } (y'/x')/\alpha(x, y)\right]$. (Received November 30, 1936.)

42. Dr. A. J. Maria: *Equilibrium point of Green's function for a spherical shell.*

Let O be the center of the spheres of radii R_1 and R_2, $R_1<R_2$, bounding the shell Σ; let the pole of the Green's function for Σ be at a distance r_0 and the corresponding equilibrium point at a distance r from O. The principal result of this article is that $R_1<\alpha<r<\beta<R_2$ when $R_1<r_0<R_2$. (Received November 30, 1936.)

43. Professor C. N. Moore: *On the regularity of methods of summation of multiple series.*

By the use of various general theorems on convergence factors, recently obtained, for convergent and restrictedly convergent multiple series, necessary and sufficient conditions are derived for the regularity of definitions of summability in the case of multiple series. Criteria are obtained both for convergence factor definitions and sequence-transformation definitions, and the case of restricted summability is dealt with as well as the case of summability in the general sense. (Received November 30, 1936.)

44. Professor R. L. Moore: *A characterization of a compact continuum with no essential continuum of condensation.*

It is proved that in order that the compact continuum M should have no essential continuum of condensation it is necessary and sufficient that for every

two points X and Y of M there exist an uncountable collection G of mutually exclusive point sets each consisting of either one or two points of M and each separating X from Y in M. (Received December 1, 1936.)

45. Professor R. L. Moore: *Concerning the open subsets of a plane continuum.*

W. T. Reid has shown (this Bulletin, vol. 41 (1935), pp. 684–688) that if K is a proper subcontinuum of a plane continuum M then K contains a limit point of some component of $M-K$. It is shown that (1) the above proposition does not remain true if the word "subcontinuum" is replaced by "closed subset" and (2) if, in a plane, K is a closed point set and G is a countable set of mutually exclusive continua and G^* (the sum of all the continua of the set G) has no point in common with K and G^*+K is a continuum, then every element of G is a component of G^*. (Received December 1, 1936.)

46. Professor Marston Morse: *Homotopic extremals.*

This paper continues with the systematic development of ideas presented at various times during the last two years. We are concerned with an abstract variational theory based on a study of a function F lower semi-continuous on an abstract space M. The theory has two principal parts: (I) the analysis of the distribution of critical values by group theoretic means, involving indexed subsets of additive, abelian, operator groups, and (II) a local topological definition of a critical point. We are here concerned with (II); and, of four different types of definitions of critical points, homotopic, combinatorial, and so on, which we have developed, we select the first. In the case where M is compact and F continuous a point p is termed *homotopically ordinary* if there exists a continuous deformation of some neighborhood of p which displaces p, and under which F is decreased whenever a point is displaced. A point not homotopically ordinary is termed critical. In the case where M is not compact and F merely semi-continuous we make the necessary technical modifications. Variational problems lead to spaces M in which the "point" is a curve, and the homotopic point is then termed a homotopic extremal. The fundamental theorem here is as follows. When the metric M is derived from the study of ordinary variational problems with a positive integrand on a compact differentiable space of class C^4 each homotopic extremal is an ordinary extremal. This theorem includes the Hilbert theorem that the absolute minimizing curve is an extremal, and it assumes nothing concerning the differentiability of the homotopic extremal. (Received December 1, 1936.)

47. Professor Rufus Oldenburger: *Real canonical binary trilinear forms.*

In earlier papers Dedekind, Gilham, others, and the author treated the problem of equivalence of binary trilinear forms under the class of non-singular linear transformations in the complex field. In the present paper there are obtained the canonical forms to which binary trilinear forms are equivalent under non-singular linear transformations in the field of reals and a complete invariant system. Five canonical forms are obtained, one more than for the complex field. The reductions to four canonical forms are given, while it is proved by the

existence of roots of certain inequalities that forms not equivalent to these four are equivalent to the fifth. The complete invariant system obtained is of a rather simple character, comprising ranks defined in terms of 3-way matrices and properties of associated invariant factors. (Received November 27, 1936.)

48. Professor E. G. Olds: *Distributions of sums of squares of rank differences for small numbers of individuals.*

In a recent article (Annals of Mathematical Statistics, vol. 7 (1936), pp. 29–43) Hotelling and Pabst have indicated the need for the exact distribution of the coefficient of rank correlation for small values of n. From the article we have $i=1-6\sum d^2/(n^3-n)$, where i is the rank correlation coefficient, n the number of individuals, and $\sum d^2=\sum_{i=1}^{n} d_i^2$ (d_i being the rank difference for the ith individual). For n fixed in value, the distribution of i depends on the distribution of $\sum d^2$. This paper will be concerned with (1) the exact distribution of $\sum d^2$ for certain small values of n, (2) approximations obtained by using ordinates of the Pearson type II curve, (3) approximations obtained by using the normal curve, (4) the construction of a table to test the significance of rank correlation in small samples. (Received November 18, 1936.)

49. Professor Oystein Ore: *Extensions of the theorem of Jordan-Hölder in groups.*

It is shown how the theorem of Jordan-Hölder and the refinement theorem of Schreier-Zassenhaus may be extended to certain non-normal chains of subgroups. First a weak theorem is proved for "permutable" chains and next a stronger theorem by means of a new concept of quasi-normal subgroups. (Received November 14, 1936.)

50. Dr. J. F. Randolph: *Metric separability and the Hildebrandt integral.*

Two point sets are "metrically separated" if the outer measure of their sum is the sum of their outer measures (when each has outer measure finite). A function is "metrically separable" on a set A if for every constant the part of A where the function is greater than a constant is metrically separated from the rest of A. All measurable functions are metrically separable, but not conversely. It has been shown by Jeffery (Annals of Mathematics, vol. 33, pp. 443–459) that metric separability may be used as a basis for a comprehensive theory of integration (as suggested by Hildebrandt, this Bulletin, vol. 24 (1917), p. 128) which includes as special cases Young, Pierpont, and Lebesgue integration. Similar results for integration over plane sets with respect to Carathéodory outer linear measure do not follow from Jeffery's methods. The present paper develops new methods which envelop these broader applications. (Received December 1, 1936.)

51. Professor J. H. Roberts: *A metrization theorem.*

In the present paper it is proved that if S is a metric space and G is an upper semi-continuous collection of closed sets filling S (R. L. Moore's topological

definition) then the space determined by the elements of G with the usual definition of limit element is metrizable. Furthermore if S is complete so is G. These results are obtained as consequences of the following new metrization theorem: Suppose $L_1(x, y)$ and $L_2(x, y)$ are functions defined for every x, y in a topological space E, and $L_i(x, y)$, $(i=1, 2)$, satisfies all the postulates for a distance function except the triangle inequality. Suppose furthermore that for any x, y, and z in E, $L_1(x, y)+L_1(y, z) \geqq L_2(x, z)$. Then E is metrizable. (Received November 23, 1936.)

52. Dr. S. L. Robinson: *Pseudo-k-fold transitive groups.*

A permutation group of degree n is pseudo-k-fold transitive if an unordered set of k letters can be changed into every other such unordered set by permutations of the group. In case n is large compared with k, it is shown that a pseudo-k-fold transitive group is $k-1$ times transitive. (Received December 1, 1936.)

53. Dr. A. R. Schweitzer: *Definitions of betweenness in the foundations of geometry.*

Dr. Schweitzer constructs two categories of definitions of betweenness in descriptive geometry (1) with reference to orientation and (2) primarily without reference to orientation. In the first category the author has given (American Journal of Mathematics, 1909) definitions of betweenness in terms of his relations R and K. Definitions of betweenness in the second category and associated descriptive systems of axioms are given in the present paper. These definitions are phrased for n equal to 1, 2, 3, . . . , in terms of the relations S_n, B_n, and I_n (loc. cit., p. 366). Definitions in terms of the relations S_n and B_n are constructed in analogy with those given by the author in terms of the relations R_n and K_n. Betweenness in terms of the relation I_n is defined in two ways, first in analogy with definitions of the oriented type and second in terms of the (defined) concept of a point lying on the boundary of an n-dimensional $n+1$ point. In the case of the relation I_n transition to metrical geometry is made by assuming that $n+1$ independent points are inscribed in a circle, sphere, hypersphere, . . . , the latter concepts being undefined under the axioms (see the reference to cyclic and spherical order, loc. cit., pp. 385, 389). Metrical definitions of betweenness have been given by Pieri, Huntington, and others. (Received November 25, 1936.)

54. Professor J. A. Shohat: *Application of Laguerre polynomials to Laplace integrals.*

Laplace integrals and Lorch's theorem are discussed by means of Parseval's formula for Laguerre polynomials. (Received December 1, 1936.)

55. Dr. Alvin Sugar: *A note on James' asymptotic Waring theorem for the cubic polynomial.*

By employing a transformation, the congruential condition of James' theorem is made less restrictive. (Received November 27, 1936.)

56. Professor Otto Szász: *On the absolute convergence of Fourier series.*

In 1934 S. Bernstein proved the theorem: Let $f(\theta)$ be a continuous periodic function of period 2π. Let c_n be the Fourier constants of $f(\theta)$. Define $\xi_f(\delta)$ by $\xi_f(\delta)=\max_{|\theta|\leq\pi,\,|t|\leq\delta}|f(\theta+t)-f(\theta)|$, $\delta>0$; then, if $\sum_{n=1}^{\infty}n^{-1/2}\xi_f(1/n)<\infty$, it follows that $\sum_{n=1}^{\infty}|c_n|<\infty$. Conversely, given a function $\eta(\delta)$ for which $\eta(\delta)/\delta^{\alpha}$ increases for sufficiently small α's and decreases for a certain $\alpha<1$, as δ increases; if, besides, the series $\sum n^{-1/2}\eta(1/n)$ diverges, then there exists a function $f(\theta)$ such that $\xi_f(\delta)<\eta(\delta)$ and $\sum|c_n|$ diverges. In this paper this result is generalized as follows: Let $f(\theta)$ be a function in L^p where $1<p\leq 2$ with the period 2π. Define $\omega_f(\delta)$ by $\omega_f(\delta)=\max_{|t|\leq\delta}[(1/2\pi)\int_{-\pi}^{\pi}|f(\theta+t)-f(\theta)|^p d\theta]^{1/p}$. Then, if $\sum n^{-(p-1)/p}\omega_f(1/n)<\infty$, it follows that $\sum|c_n|<\infty$. Conversely, given a function $\gamma(\delta)$ of a certain type for which $\sum n^{-(p-1)/p}\eta(1/n)$ diverges, then there exists a function $f(\theta)$ for which $\omega_f(\delta)<\eta(\delta)$ and such that $\sum|c_n|$ diverges. The case $p=2$ includes Bernstein's theorem. In the proof of the first part, the inequalities of Hausdorff and Hölder are used. For the proof of the second part certain relations between the degree of approximation and the "modulus" of continuity in L^p arise. (Received November 23, 1936.)

57. Professor J. M. Thomas: *Functional dependence of polynomials.*

If $f_1, \cdots, f_{k+1}$ belong to the commutative polynomial ring $R[x_1, \cdots, x_n]$ and the rank of their functional matrix is k, a method is given for constructing in $R[f_1, \cdots, f_{k+1}]$ a non-zero polynomial $F(f_1, \cdots, f_{k+1})$, which vanishes as a member of $R[x_1, \cdots, x_n]$. Thus the function whose existence is stated by the dependence theorem of analysis can be taken as a polynomial when the given functions are all polynomials. (Received December 1, 1936.)

58. Professor H. S. Thurston: *On the irregular case of the quadratic equation in quaternions and binary matrices.*

Sylvester (Comptes Rendus, vol. 99, pp. 555–558, 621–631) showed that the equation $X^2+PX+Q=0$, where X, P, Q, are quaternions (or binary matrices), has six solutions, except when a certain cubic equation in a scalar variable, λ, has a root $\lambda=0$. This case he termed the *irregular* case. In this paper, a necessary and sufficient condition for the occurrence of the irregular case is obtained. It is found that the irregular case occurs if P and Q are linear functions of a quaternion (or binary matrix) A; in the quaternion equation this condition is also necessary. The paper discusses the number and nature of the solutions when the condition is satisfied. (Received December 1, 1936.)

59. Mr. R. E. Traber: *On simple convex neighborhoods.*

This paper presents a simpler proof of the existence of simpler convex neighborhoods in the geometry of paths, a theorem first proved by J. H. C. Whitehead (Quarterly Journal of Mathematics, vol. 3 (1932), pp. 33–42; vol. 4 (1933), pp. 226–227). It is based on a theorem on the domain of normal coordinates with origin at a point in a compact set, and uses the well known point-

direction existence theorem on ordinary differential equations instead of the more involved two point form used by Whitehead. (Received November 27, 1936.)

60. Professor W. J. Trjitzinsky: *Analytic theory of non-linear singular differential equations.*

In this memoir, which will appear in the Mémorial des Sciences Mathématiques, the author develops an analytic theory of the non-linear differential equation of order n: $x^p y^{(n)}(x) = b(x, y, y^{(1)}, \cdots, y^{(n-1)})$, where p is a positive integer, and where the second member is analytic in the displayed arguments at $(x=0, y=y^{(1)}= \cdots =y^{(n-1)}=0)$. When b is linear in $y, y^{(1)}, \cdots, y^{(n-1)}$, we have a linear problem with a singularity of finite rank at $x=0$. The general analytic theory of equations of the latter type has been previously developed by the author of the present paper (Acta Mathematica, vol. 62, pp. 167–226). In the present work properties of solutions of the differential equation stated above are investigated in the complex neighborhood of the singular point $x=0$. (Received November 30, 1936.)

61. Professor A. W. Tucker: *Tensor algebra in topology.*

The algebra of the mappings and products of abstract complexes can be formulated advantageously in tensor terms. Correspondence numbers, or alternatively the coefficients of product chains, provide the tensor components. The boundary operator yields a sort of absolute differentiation. The relations between a complex and its dual reflect themselves of course in covariance and contravariance. The heart of the algebra is a non-commutative (essentially skew-commutative) contraction process which generalizes the notion of Kronecker intersection; inherent in this are a variety of products. (Received December 1, 1936.)

62. Dr. H. E. Vaughan: *On locally bicompact spaces.*

Two classes of locally bicompact spaces are defined each of which is a generalization of the class of locally compact separable metric spaces in the sense that each contains every space of the latter class and contains no other metric space although, in general, the spaces defined are not separable. Two additional characterizations of each class are obtained, one in terms of the covering theorems which hold in the spaces in question, the other in terms of the complements of the spaces with respect to the bicompact spaces in which they can be imbedded. It is shown that the spaces of one class are normal, while those of the other class are completely normal, and that many of the properties usually derived from perfect separability belong to the spaces of the latter class. (Received December 1, 1936.)

63. Professor J. L. Walsh: *On the shape of level curves of Green's function.*

If the point O lies interior to the Jordan curve C, we define the *circularity* of C as the quotient $K(C)$ of the shortest distance from O to a point of C by the longest distance from O to a point of C. Under a smooth conformal map $z=\psi(w)$ of the interior of the unit circle $|z|<1$ onto the interior of a Jordan

curve in the w-plane, the circularity of C_r: $|\psi(w)| = r \leqq 1$ is non-decreasing as r decreases, approaches unity as r approaches zero, and satisfies the inequality $K(C_r) \geqq [K(C_1)]^{(4/\pi)\tan^{-1}r}$. (Received November 25, 1936.)

64. Professor Norbert Wiener: *A class of Tauberian theorems.*

Professor Wiener uses the method of undetermined functions to reduce a class of Tauberian theorems dealing with power series of rapid growth to the form where the kernel is a function of the difference of two variables. The functional equations thus derived are easily solvable. Among the results are certain gap theorems concerning power series of very rapid growth. It turns out that if this growth is sufficiently smooth the gap for the nth term cannot be of much greater order than the square root of n. (Received November 18, 1936.)

65. Professor F. L. Wren: *A theorem on determinant expansion and some applications.* Preliminary report.

A theorem on the expansion of determinants, which is a generalization of a theorem by Sylvester and also one by Chió, is established. This theorem is then applied to the solution of systems of linear equations, determining the rank of matrices, and the calculation of partial and multiple coefficients of correlation. (Received November 30, 1936.)

66. Professor Oscar Zariski: *Generalized weight properties of the resultant of $n+1$ polynomials in n indeterminates.*

Let $f_1, \cdots, f_{n+1}$ be $n+1$ polynomials in $x_1, \cdots, x_n$, of degrees $l_1, \cdots, l_{n+1}$ respectively, and let $s_1, \cdots, s_{n+1}$ be integers, $0 \leqq s_i < l_i$. The weight of any coefficient of f_i is defined as $s_i - j_i$, or zero, according as $j_i \leqq s_i$ or $j_i \geqq s_i$, where j_i is the degree of the corresponding term of f_i. It is proved that each term of the resultant $R = R(f_1, \cdots, f_{n+1})$ is of weight $\geqq s_1 s_2 \cdots s_{n+1}$. Moreover, if ϕ_i and ψ_i denote the sum of terms of f_i of degree $\leqq s_i$ and of degree $\geqq s_i$ respectively, then the sum of terms in R which are of weight $s_1 s_2 \cdots s_{n+1}$ is given by the expression $D^\sigma R(\phi_1, \cdots, \phi_{n+1})$, where D is the base of the (principal) ideal of the inertia forms of $\psi_1, \cdots, \psi_{n+1}$. Here $\sigma = 1$, except when all $s_i = l_i$ but one, say s_1, in which case $\sigma = l_1 - s_1$. These properties of the resultant lead to a direct algebraic derivation of the intersection multiplicity λ of $n+1$ hypersurfaces F_i in S_{n+1} at a common point P, of multiplicity s_i for F_i: $\lambda \geqq s_1 s_2 \cdots s_{n+1}$ and $\lambda = s_1 s_2 \cdots s_{n+1}$ if and only if the hypersurfaces have no common tangent at P. (Received December 1, 1936.)

67. Professor Oscar Zariski: *On the behavior of zero-dimensional ideals under quadratic transformations.*

A singular point P of an algebraic variety V in a linear space $S_n(x_1, x_2, \cdots, x_n)$ can be described by means of a suitable primary 0-dimensional ideal $\mathfrak{A}$ in the polynomial ring $K[x_1, \cdots, x_n]$, where K is the underlying field of constants. To resolve the singularity at P (assuming P to be the origin) it is customary to employ the quadratic transformation: $x_1' = x_1$, $x_i' = x_i/x_1$, $i = 2, \cdots, n$. That the singularity is actually simplified by the transformation is well known in the case of algebraic curves and in the case of surfaces in S_3—

and in these cases only. Proceeding in a purely arithmetic fashion, the author considers the length of the ideal $\mathfrak{A}$ as a controlling character of the singularity. Given an arbitrary primary 0-dimensional ideal $\mathfrak{A}$ in $K[x_1, \cdots, x_n]$, not necessarily defined by a singular point, a comparison is made of the length of $\mathfrak{A}$ with the length of each of the primary components $\mathfrak{A}_i'$ of the extended ideal of $\mathfrak{A}$ in the larger polynomial ring $K[x_1', \cdots, x_n']$. It is found that the length of each $\mathfrak{A}_i'$ is definitely less than the length of $\mathfrak{A}$ only if $n \leqq 3$, but that if $n > 3$, the length may actually be increased by the transformation. The positive part of the result furnishes at any rate an arithmetic proof for the resolution of the point singularities of algebraic surfaces. In view of the negative result in the case $n > 3$, it appears that a similar proof for higher varieties must take into consideration the special character of the ideals defined by point singularities. (Received December 1, 1936.)

68. Professor Oscar Zariski: *The topological discriminant group of a Riemann surface of genus p and an application to the Poincaré group of plane elliptic curves.*

By the *topological discriminant group of degree n* of a complex K is meant the Poincaré group of $K^n - D$, where K^n is the symmetric nth product of K and D is the subcomplex of K^n representing the n-tuples with two or more coincident points. This group, G_n, is explicitly determined when K is a Riemann surface of genus p (for $p = 0$, see the author's paper in the American Journal of Mathematics, 1936). In the case $p = 1$ it is shown that the commutator group of G_n gives the Poincaré group of the maximal cuspidal plane elliptic curve of order $2n$. This result is applied toward the determination of the Poincaré group of an arbitrary plane elliptic curve with cusps and nodes. The special case $n = 3$ gives rise to a topological interpretation of the properties of the configuration of the flexes of a plane elliptic cubic. (Received December 1, 1936.)

69. Dr. J. A. Greenwood: *Restricted absolute permutations.*

The problem is equivalent to finding the number of non-vanishing terms and value of a determinant of ones and zeros of order n having n zeros down the principal diagonal and s other zeros subject only to the restriction that every principal first minor have $\geqq n+s-2$ zeros. It is found that the number of terms $N_n(s) = \sum_{i=1}^{s} (-1)^i C_{s,i} N_{n-i}^{n-2i}$, where the numbers $N_m^q = \sum_{r=0}^{q} (-1)^r C_{q,r}\,(m-r)!$ were evaluated by Longchamps (1891). The value of the determinant is found to be $(-1)^n \cdot (1+s-n)$. From these two results the exact number of even and odd permutations of this restricted absolute set is easily obtained. This is a generalization of the case $s = 0$ published by J. M. Thomas in this Bulletin, 1925. The result, $\lim_{n\to\infty} N_n(0)/n! = \exp(-1)$, which is due to Cantor, is now generalized to $[\exp(-1) \geqq \exp(-1-\underline{\phi(n)/n}) = N_n[\phi(n)]/n! \geqq \exp(-1-\overline{\phi(n)/n}) \geqq \exp(-3/2)]_{(n\to\infty)}$. (Received November 27, 1936.)

70. Mr. I. Halperin and Professor John von Neumann: *On the transitivity of perspective mappings in complemented modular lattices.*

Transitivity is an essential property of the equivalence by perspectivity in continuous geometries (see J. von Neumann, Proceedings of the National

Academy, 1936, pp. 92–101). Its proof is rather complicated, and uses the transcendent "continuity axioms" of this theory. The general transitivity is established by proving it first for non-intersecting elements. In the present note a simpler proof of this last fact is given, a great part of the proof being quite elementary and independent of the "continuity axioms," hence valid for all complemented modular lattices. (Received December 2, 1936.)

71. Mr. I. Halperin: *On the transitivity of perspectivities in continuous geometries.*

The transitivity of perspectivity (see abstract 43-1-70) is established by a simpler method, and without using the irreducibility of the continuous geometry. The additivity and continuity properties of equivalence by perspectivity are also obtained more briefly. (Received December 2, 1936.)

72. Dr. Olaf Helmer: *Polynomials in infinitely many variables.*

Let $x_1, x_2, \cdots, x_n, \cdots$ be a sequence of infinitely many variables, running through all sequences $a_1, a_2, \cdots$ for which $\sum |a_n|$ converges. $f(x_1, x_2, \cdots)$ is called a *polynomial* in $x_1, x_2, \cdots$ if, roughly, it is a polynomial in the ordinary sense in each of the variables x_1, x_2, and so on. Thus, $x_1+x_2^2+x_3^3+\cdots$ and $x_1+x_1x_2+x_1x_2x_3+\cdots$ are polynomials in this sense, but $x_1+x_1^2x_2+x_1^3x_3+\cdots$ is not. The symbols 1, 2, 3, $\cdots$, taken in any order of the ordinal type ω, are said to form a *permutation* of 1, 2, 3, $\cdots$. Let the notation be similar to the customary one: $P=\binom{1\ 2\ 3\ \cdots}{a_1 a_2 a_3\ \cdots}$. The function resulting from the application of P to the arguments of $f(x_1, x_2, \cdots)$ may be designated by $f_P(x_1, x_2, \cdots)$. The latter is a polynomial if, and only if, the former is one. f is called *symmetrical*, if $f_P=f$ for every P. The *elementary symmetrical functions* can now be defined as usual; they are polynomials in our sense. The *main theorem on symmetric functions* can be extended to polynomials as defined here. Most of the usual proofs fail, as they involve complete induction with respect to the number of variables. The Gauss-Waring proof, however, can be so modified as to fit the requirements. (Received December 1, 1936.)

73. Dr. Nathan Jacobson (National Research Fellow): *Abstract derivation and Lie algebras.*

Let $\mathfrak{A}$ be an arbitrary algebra (not necessarily associative or commutative or of finite order) over a commutative field. We define a *derivation* D in $\mathfrak{A}$ as a linear mapping of $\mathfrak{A}$ into itself such that $(xy)D=(xD)y+x(yD)$. The set of derivations is shown to be a Lie algebra $\mathfrak{D}$ (with a slight restriction when $\mathfrak{A}$ has characteristic $\neq 0$). In the first part of this paper we discuss some elementary properties of $\mathfrak{D}$. Next we determine $\mathfrak{D}$ for a semi-simple associative algebra with a finite basis. A fundamental result is that if $\mathfrak{A}_1$ is a semi-simple subalgebra of a normal simple algebra, then any derivation in $\mathfrak{A}_1$ may be extended to an *inner* derivation in $\mathfrak{A}$, that is, a derivation of the form $x\rightarrow xd-dx\equiv xD$. Some of the algebras obtained in the case of characteristic $\neq 0$ are semi-simple Lie algebras which are not direct sums of simple algebras. If $\mathfrak{A}$ is an inseparable field, $\mathfrak{D}$ is a simple algebra of a type which has no counterpart for characteristic 0. We discuss this algebra and its relation to $\mathfrak{A}$ in greater detail, obtaining a

number of results analogous to well known theorems on automorphisms of fields. For example, we give a characterization of the logarithmic derivatives of $\mathfrak{A}$ which is analogous to Hilbert's theorem on the elements of norm 1 in a cyclic field. (Received December 1, 1936.)

74. Dr. Nathan Jacobson (National Research Fellow): *A class of normal simple Lie algebras of characteristic* 0.

Let $\mathfrak{A}$ be an involutorial normal simple algebra over a field Φ of characteristic 0 and J an involutorial anti-automorphism in $\mathfrak{A}$. The set $\mathfrak{S}_J$ of *J-skew* elements $a(a^J=-a)$ is closed under addition, scalar multiplication, and commutation. We show that $\mathfrak{S}_J$ but for one trivial exception is a simple Lie algebra which becomes either the complex or orthogonal Lie algebras when Φ is extended to its algebraic closure Ω. $\mathfrak{S}_{J_1}$ and $\mathfrak{S}_{J_2}$ are isomorphic if and only if the associative algebras $\mathfrak{A}_1$ and $\mathfrak{A}_2$ defining them are isomorphic and J_1 and J_2 are *cogredient* in the sense that $J_1=S^{-1}J_2S$ where S is an isomorphism of $\mathfrak{A}_2$ into $\mathfrak{A}_1$. The automorphisms of $\mathfrak{S}_J$ are realized as transformations $a\to g^{-1}ag$ where g is *J-orthogonal* ($g^Jg=\gamma_1\neq 0$ in Φ). Any Lie algebra $\mathfrak{L}$ over Φ such that $\mathfrak{L}_\Omega$ is isomorphic to the complex or orthogonal Lie algebras may be realized as an algebra $\mathfrak{S}_J$. The question of cogredience of anti-automorphisms is essentially one of ordinary cogredience of hermitean and skew-hermitean matrices with elements in a normal division algebra. We hope to consider this problem in a later paper. (Received December 1, 1936.)

75. Professor E. J. McShane: *A navigation problem in the calculus of variations.*

An airplane is to travel from one point to another in the least possible time. Its velocity relative to the air is required to lie in a convex set $K(x, t)$ of vectors depending on the place x and the time t. The air velocity is a continuous vector function $u(x, t)$. Existence theorems are established for this problem and for the modified form of the problem in which the velocity is required to be almost always as great as possible for the time, place, and direction of the airplane. This is a generalization of the Zermelo navigation problem. (Received December 2, 1936.)

76. Professor E. J. McShane: *Jensen's inequality.*

Jensen's inequality is established in the following general form. If L is a linear class of functions $f(x)$ and Mf is a linear mean-value functional on L such that $M1=1$ and $Mf\geqq 0$ if $f\geqq 0$, then every convex function $\phi(z_1, \cdots, z_n)$ satisfies the inequality $\phi(Mf_1, \cdots, Mf_n)\leqq M\phi(f_1, \cdots, f_n)$. Conditions for equality are investigated. (Received December 2, 1936.)

77. Dr. M. S. Robertson: *A representation of all analytic functions in terms of functions with positive real part.*

Any analytic function by trivial transformations can be normalized to the form $f(z)=z^k+\sum_{k+1}^{\infty}a_nz^n$, $k\geqq 1$, holomorphic for $|z|<1$ with the property that for each $r<1$ the map of $|z|=r$ is a contour cut by the real axis in exactly $2k$ points. It is shown how such a function can be completely characterized in

terms of a function with positive real part in the unit circle and $2k$ parameters θ_i whose values depend upon $f(z)$. From this representation one deduces that $|a_n| \leqq (n/k)\cdot(n+k-1)!/(2k-1)!(n-k)! = O(n^{2k})$ where the equality sign is attained by essentially only one function of this class. If $f(z)$ is also real on the real axis $a_n = O(n^{2k-1})$. If $g(z) = z + \sum_2^\infty c_n z^n$ is regular for $|z| < 1$ and star-like in $2k$ symmetric directions with respect to the origin (but not necessarily univalent) then $c_n = O(n^{1+(1/k)})$. For the functions $f(z)$ and $g(z)$ the radial limits $(r \to 1)$ exist finitely almost everywhere. (Received November 30, 1936.)

78. Dr. W. E. Sewell: *Degree of approximation by polynomials —problem α.*

Let C be a Jordan curve in the z-plane and let $z = \psi(w)$ map the exterior of C on $|w| > 1$, so that the points at ∞ correspond to each other. Let $\psi''(w)$ be continuous in $|w| \geqq 1$. *Let $f(z)$ be analytic in C, and satisfy a Lipschitz condition of order α, $0 < \alpha \leqq 1$, in $\overline{C}$, the closed limited point set bounded by C. Then there exist polynomials $P_n(z)$, $n = 2, 3, \cdots$, of degree n in z, such that $|f(z) - P_n(z)| \leqq M (\log n/n)^\alpha$, z in $\overline{C}$, where M is a constant independent of n and z.* (Received December 1, 1936.)

79. Professor D. J. Struik: *On surface theory in four space.*

Different possibilities of generalizing the ordinary surface theory to four space are sketched, with special reference to developable surfaces and ruled surfaces in general. An application is given to certain surfaces of zero curvature. (Received December 1, 1936.)

80. Professor G. T. Whyburn: *On interior transformations.*

Let $T(A) = B$ be an interior transformation, where A is compact and metric. Then (i) for any subset Q of A satisfying $Q = T^{-1}T(Q)$, T is an interior transformation on Q; (ii) for any continuum $C \subset B$, every component of $T^{-1}(C)$ transforms into the whole of C; (iii) for any open set R in A, $F[T(R)] \subset T[F(R)]$, where $F(X)$ denotes the boundary of X; (iv) if A is locally connected, B_0 is any closed set in B, and Q is any component of $B - B_0$, then $T^{-1}(Q)$ is the sum of a finite number of components of $A - T^{-1}(B_0)$ each of which maps into exactly Q. These results yield a complete analysis of interior transformations defined on a number of particular types of sets A. For example, if A is a simple closed curve, B must be either a simple closed curve or an arc; and in the former case T is completely alternating and hence is equivalent to the transformation $w = z^k$ on $|z| = 1$, while in the latter, T is equivalent to the transformation $f(x) = \cos k\pi x$ of $(0, 1)$ into $(-1, 1)$. (Received January 5, 1937.)

81. Dr. J. H. Chanler: *The involution curve determined from a special pencil of n-ics.*

The author investigates the involution curve C_k determined by k-ads from a pencil of binary n-ics which contains one member with j double roots. The study was suggested by the curves $[W_p, S_p]$ of W_p (A. B. Coble and J. H. Chanler, *The geometry of the Weddle manifold W_p*, American Journal of Mathematics, vol. 57 (1935), pp. 183–218). The genus and multiple points of C_k are

determined by applying Zeuthen's formula to the correspondence set up between the points of C_k and the points of the norm curve. (The method of counting branch points used in §§2, 6 is due to Professor O. Zariski.) The curve and manifolds associated with it are also studied from the projective viewpoint. By this means manifolds investigated by Brill are proved to be special cases of those studied by Salmon, Cayley, and others. Lastly cases are discussed in which some of the j double roots of the special n-ic coincide. (Received December 7, 1936.)

82. Professor N. A. Court: *Desmic tetrahedrons related to quadric surfaces.*

Given the tetrahedron (T) inscribed in the sphere (S), A. A. Bennett determined analytically the tetrahedron (K) self-polar both with respect to (S) and (T) (American Mathematical Monthly, 1932, pp. 18–27). A synthetic discussion of this problem, in which the sphere (S) is replaced by a non-ruled quadric (Q), leads to the following construction of (K). Let a, a' be a pair of opposite edges of (T), x, x' their polar lines with respect to (Q), and u, u' the two lines meeting the lines a, a', x, x'. The lines u, u' are unique and are always real. If K, K' are the double elements of the involution determined by the two pairs of points ua, ua'; ux, ux', and K'', K''' the two analogous points on u', the points K, K', K'', K''' are the vertices of (K). The tetrahedron (T'), which with (T) and (K) forms a desmic system, is inscribed in (Q) and is the only tetrahedron inscribed in (Q) and self-polar with respect to (T). (Received December 3, 1936.)

83. Dr. S. C. Kleene: *On notation for ordinal numbers.* Preliminary report.

There exists a *greatest* ordinal ξ in the second number class such that there is a system of notation in which (1) at least one formula X is assigned to represent each ordinal $x<\xi$, (2) no X represents two distinct ordinals, (3) from X it is effectively decidable whether $x=0$ or $x=y+1$ or $x=\lim_r\{x_r\}$, (4) if $x=y+1$, then from the formula X a formula Y representing y is effectively calculable, (5) if $x=\lim_r\{x_r\}$, then from X a sequence of formulas $x_1, x_2, \cdots$ representing $x_1, x_2, \cdots$, respectively, is effectively calculable. (Received December 29, 1936.)

84. Professor R. G. Lubben: *Concerning perfectly normal Hausdorff spaces.*

1. In order that a Hausdorff space should be perfectly normal, it is necessary and sufficient that each closed set in it be both the product and the limiting set of a countable collection of open sets. 2. In order that a Hausdorff space having the Lindelöf property should be perfectly normal it is necessary and sufficient that it be regular. 3. In a Hausdorff space having the Lindelöf property, in order that every point set should have the Lindelöf property it is sufficient that the space be perfectly normal. (Received December 29, 1936.)

85. Professor H. A. Rademacher: *On the partition function.*

In 1917 Hardy and Ramanujan gave an asymptotic formula for the number $p(n)$ of unrestricted partitions of n. A revision and extension of their method

leads to a convergent series for $p(n)$, of which the asymptotic formula is an immediate consequence. (Received December 21, 1936.)

86. Mr. L. B. Robinson: *A system of Riquier and the tensor calculus.* II.

The complete system (1) $\sum_{j=1}^{r} P_{ij}(x_1 x_2 \cdots x_j)\partial f/\partial x_{j+1}=0$, $(i=1, 2, \cdots, n)$, where the P are polynomials, shall be called a system of class K. Solve the final equation of the system by quadratures. The solutions $w_2 \equiv x_1, w_3, w_4, \cdots, w_{r+1}$ can serve as new variables and can be substituted in the remaining equations, $n-1$ in number. The author now has a new system $\sum_{j=2}^{r} Q_{ij}(w_2, w_3, \cdots, w_j) \cdot \partial f/\partial w_{j+1}=0$ which is also of class K. Continue and solve the entire system by quadratures. The final solution is written: $v_1 \equiv x_1$, $v_2 \equiv x_{n+2}+f_{n+2}(x_1, x_2, \cdots, x_n, x_{n+1})$, $v_3 \equiv x_{n+3}+f_{n+3}(x_1, x_2, \cdots, x_n, x_{n+1}, x_{n+2}), \cdots, v_{r+1} \equiv x_{r+1-n}+f_{r+1}(x_1, x_2, \cdots, x_r)$ where the f are rational with respect to x_1 and rational and integral with respect to the other variables. In next resumé the author plans to extend these results to a more general class and then apply the theorems to the tensor calculus. (Received December 29, 1936.)

87. Professor D. V. Widder: *The Stieltjes transform.*

This paper is concerned with the transform $f(x)=\int_0^\infty (x+t)^{-1}d\alpha(t)$. A real inversion formula analogous to the Taylor determination of the coefficients of a power series is obtained. We define $L_{k,t}[f(x)]$ as a linear differential operator whose fundamental solutions are $x^n (n=-k, -k+1, \cdots, -1, 0, 1, \cdots, k-2)$. If $\alpha(t)$ is the integral of a function $\phi(t)$, then $L_{k,t}[f(x)]$ approaches $\phi(t)$ for almost all positive values of t. If $\alpha(t)$ is a normalized function of bounded variation, the integral of this same operator between the limits 0 and t approaches $\alpha(t)-\alpha(0+)$ for all positive t. Necessary and sufficient conditions on $f(x)$ that it should be the transform of a function $\alpha(t)$ are obtained. Finally the relation of the above inversion formula to one given recently by Paley and Wiener is investigated. Symbolically, the latter is $(\pi)^{-1}t^{-1/2}(\cos \pi D)(t^{1/2}f(t))$, where D is the operation of differentiation with respect to t. Paley and Wiener use the power series development of the cosine, whereas $L_{k,t}[f]$ is a section of the familiar infinite product development of the cosine. (Received December 15, 1936.)

88. Professor E. L. Dodd: *Internal and external means arising from the scaling of frequency functions.*

It is well known that a scale obtained in a curve-fitting process is sometimes a mean. For example, with the normal function, constant $\cdot \exp\{-(x/a)^2/2\}$, the scale a is usually taken as the root-mean-square of the given measurements. The R. A. Fisher maximum likelihood formula, indeed, leads to just this scale. In scaling by the Fisher method, however, the usual criterion sometimes leads to minimum likelihood instead of to maximum likelihood. And scaling is not always unique—sometimes a scale with minimum likelihood will be an internal mean; and a scale with maximum likelihood, an external mean. Here a generalized mean $F(x_1, x_2, \cdots, x_n)$ is used, subject to the restriction that if c is a value which the x's can take on, then $F(c, c, \cdots, c)=c$. Using the likelihood

method, conditions are given (1) that a scale known as existing will be a *mean*; (2) that such a scale will be an *internal* mean as regards absolute values; and (3) that a scale will *exist*. The theory is illustrated largely from Pearson types. (Received January 4, 1937.)

89. Dr. J. M. Feld: *Plane cubic curves invariant under quadratic transformations.*

In a paper by K. Ogura, (Tôhoku Mathematical Journal, 4 (1913), p. 132), it was shown that the non-singular plane cubics invariant under the quadratic transformation $x_1' : x_2' : x_3' = x_2x_3 : x_3x_1 : x_1x_2$, fall into two classes bearing the equations A: $\sum a_1x_1(x_2^2 - x_3^2) = 0$ and B: $\sum a_1x_1(x_2^2 + x_3^2) + a_4x_1x_2x_3 = 0$. It is proved in this report that every non-singular cubic is invariant under this quadratic transformation and, moreover, that every non-singular cubic may bear equation A or B, according to the manner in which the fundamental triangle is selected. (Received December 31, 1936.)

90. Dr. Aaron Fialkow (National Research Fellow): *Systems of plane curves.*

The author studies systems of ∞^2 curves which have the Cesàro-Scheffers property that the centers of the ∞^1 circles which osculate those curves passing through a common point lie on a straight line. These two-parameter systems include the isogonal trajectories of any one-parameter family and the extremals of a common calculus of variations problem as two important types. A consideration of the envelope of any ∞^1 Cesàro-Scheffers lines leads to a projectivity. It is characteristic of the extremals that this projectivity is always an involution. This result also gives a new characterization of isothermal curves. Isogonal systems may be completely described by means of Lamé's relation. If the two-parameter family is isogonal, a simple geometric construction of the projectivity is given. In this case, the hyperosculating circles play an important role. A separate discussion is given when the totality of Cesàro-Scheffers lines degenerates into a one-parameter family. These results are applied to characterize one-parameter families of curves, $u(x, y) = c$, where $u(x, y)$ is a solution of a given partial differential equation. (Received December 17, 1936.)

91. Professor H. A. Rademacher: *A convergent series for the partition function $p(n)$.*

The method by which in 1917 Hardy and Ramanujan obtained an asymptotic formula for $p(n)$ can be modified in such a way as to give an exact expression for $p(n)$ in the form of an infinite series. This determination for $p(n)$ in turn leads to a new representation for the generating function $f(x) = \sum_0^\infty p(n)x^n$ in which its natural boundary is dissolved into distinct singularities. (Received January 7, 1937.)

92. Dr. A. R. Schweitzer: *A theory of oriented angles in the foundations of geometry.*

In verification of a conjecture made by the author in this Bulletin, vol. 15 (1908), p. 81, a system of axioms is constructed for descriptive geometry which

is equivalent to the descriptive theory of the author in the American Journal of Mathematics, vol. 31 (1909), pp. 365–411. The present system is phrased in terms of an undefined relation K expressing concretely sameness of sense of two n-dimensional angles, α_{jn} and β_{jn}, where $1 \leqq j < n (n = 2, 3, \cdots)$ and $\alpha_{jn} = (\alpha_1\alpha_2 \cdots \alpha_{n+1-j}, \alpha_1\alpha_2 \cdots \alpha_{n+2-j}, \cdots, \alpha_1\alpha_2 \cdots \alpha_{n+1})$, and similarly for β_{jn}. This descriptive theory is made complete for euclidean and lobachevskian geometry. In a second part of the paper the preceding theory of oriented angles is restated in terms of the "quotient relation," $\alpha_{jn}/\beta_{jn} = k$, where k is a real number, and is then completed in analogy with the author's theory in the American Journal of Mathematics, vol. 35 (1913), pp. 37–56. This theory of oriented angles combined with the latter theory of oriented areas, volumes, and so on, provides an adequate theory of congruence in Grassmann's extensive algebra. Metrically, the theory of angles in the foundations of geometry has been studied by E. Study (1885), D. K. Picken (1924), C. Boccallotte (1929), G. Hessenberg (1930), and others. (Received January 8, 1937.)

93. Professor W. E. Roth: *On certain matrices and their determinants.*

It is here shown that the $nr \times nr$ matrix $M = <A_0>I + <A_1>U + \cdots + <A_{r-1}>U^{r-1}$ where A_i, $(i = 0, 1, \cdots, r-1)$, are $n \times n$ matrices, where U is an $r \times r$ matrix, and where $<A_i>U^i$, $(i = 0, 1, \cdots, r-1)$, is the direct product of A_i and U^i, is equivalent to the direct sum of the r matrices $A_0 + A_1\theta_j + \cdots + A_{r-1}\theta_j^{r-1}$, $(j = 1, 2, \cdots, r)$, where θ_j are the roots of $|U - I\lambda| = 0$. As a result $|M|$ is the product of the r determinants $|A_0 + A_1\theta_j + \cdots + A_{r-1}\theta_j^{r-1}|$, $(j = 1, 2, \cdots, r)$. If the A_i are scalars, we have a generalization of the well-known results given under cyclic determinants by Pascal in *Die Determinanten*. (Received December 30, 1936.)

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

ANNUAIRE STATISTIQUE. Volume 51 (1935). Paris, Imprimerie Nationale, 1936 168 pp.

BAUSCHINGER (J.) and PETERS (J.). Logarithmisch-trigonometrische Tafeln mit acht dezimalstellen enthaltend die Logarithmen aller Zahlen von 1 bis 200,000 und die Logarithmen der trigonometrischen Funktionen für jede sexagesimalsekunde des Quadranten mit Unterstützung der preussischen Akademie der Wissenschaften in Berlin und der Akademie der Wissenschaften in Wien (Treitstiftung). Volume 1: Tafel der achtstelligen Logarithmen aller Zahlen von 1 bis 200,000. 2d edition. Leipzig, Engelmann, 1936.

BISHOP (M.). Pascal, the life of genius. New York, Reynal and Hitchcock, 1936. 11+398 pp.

CARTER (H. C.). College algebra. New York, Prentice-Hall, 1936. 10+234 pp.

COOLEY (H. R.). See GRAHAM (P. H.).

COURANT (R.). Differential and integral calculus. Volume 2. Translated by E. J. McShane. London and Glasgow, Blackie, 1936. 10+677 pp.

COWLES (W. H. H.) and THOMPSON (J. E.). A text book of trigonometry for colleges and engineering schools. New York, van Nostrand, 1936. 10+373 pp.

DAVIS (H. T.). The theory of linear operators from the standpoint of differential equations of infinite order. Bloomington, Ind., Principia Press, 1936. 14+628 pp.

DUBOURDIEU (J.). Questions topologiques de géométrie différentiaĺ. (Mémorial des Sciences Mathématiques, No. 78.) Paris, Gauthier-Villars, 1936. 63 pp.

FRÉCHET (M.). Leçons sur les séries trigonométriques. (Les Cours de la Sorbonne.) Paris, Tournier and Constans, 1936. 62 pp.

——— Théorie élémentaire des équations différentielles. Paris, Centre de Documentation Universitaire, 1936. 58 pp.

GÖLDEL (R. W.). Die Lehre von der Identität in der deutschen Logik-Wissenschaft seit Lotze. (Studien und Bibliographie zur Gegenwartsphilosophie, No. 18.) Leipzig, Hirzel, 1935. 462 pp.

GRAHAM (P. H.), JOHN (F. W.), and COOLEY (H. R.). Analytic geometry. New York, Prentice-Hall, 1936. 11+294 pp.

JOHN (F. W.). See GRAHAM (P. H.).

KRYJANOVSKY (D.). Éléments de la théorie des inégalités. Leningrad, Librairie Scientifico-technique et toutes librairies de l'URSS, 1936. 112 pp.

MCSHANE (E. J.). See COURANT (R.).

MITRA (P. N.). Spherical trigonometry. Calcutta, University of Calcutta, 1935. 22+163 pp.

PETERS (J.). See BAUSCHINGER (J.).

PHILIP (M.). Mathematical analysis. New York, Longmans Green, 1936. 11+275 pp.

RIVETT (F. A. J.). The groundwork of school geometry. London, Arnold, 1936. 128 pp.

RUPEIKA (Z.). Didziosios P. Fermat'o teoremos irodymas. Kaunas Librairie Spindulio, 1934. 32 pp.

SHIBLI (J.). Plane and spherical trigonometry, with applications. 2d edition. New York, Ginn, 1936. 12+242+94 pp.

SINGH (A. N.). The theory and construction of non-differentiable functions. (Lucknow University Studies, No. 1.) Lucknow, Newul Kishore Press, 1935. 7+110 pp.

THOMPSON (J. E.). See COWLES (W. H. H.).

TÖLKE (F.). Besselsche und Hankelsche Zylinderfunktionen nullter bis dritter Ordnung vom Argument $r\sqrt{i}$. Stuttgart, Wittwer Verlag, 1936. 92 pp.

PART II. APPLIED MATHEMATICS

APPLETON (E. V.). See JEANS (J.).

AREND (J. P.). Atombildung und Erdgestaltung. Das kausalunitarische Weltbild. With a preface by E. Haarmann. Stuttgart, Enke, 1936. 15+101 pp.

ARKADIEV (V.). Electromagnetic processes in metals. Part 2: The electromagnetic field. Moscow and Leningrad, Obedinennoe Nauchno-Tekhicheskoe Izdatelstvo Glavnaia Redaktsia Energeticheskoe Literatury, 1936. 304 pp.

BERG (E. J.). Heaviside's operational calculus: as applied to engineering and physics. (Electrical Engineering Texts.) 2d edition. New York and London, McGraw-Hill, 1936. 15+258 pp.

BÖRNSTEIN (—). See LANDOLT (—).

BOLL (M.). La chance et les jeux de hasard. Paris, Larousse, 1936. 382 pp.

BRAGG (W.). See JEANS (J.).

BRARD (R.) and GORCEIX (C.). Radiésthèsie scientifique: Balance pendulaire de précision suivie du Journal de Laboratoire de 1935. 2d edition. Paris, Lechevalier, 1935. 277 pp.

CRANZ (C.). Lehrbuch der Ballistik. Berlin, Springer, 1936. 12+292 pp.

CRENSHAW (B. H.). See SIMPSON (T. M.).

CULVER (C. A.). A textbook of physics: for students of science and engineering. New York, Macmillan, 1936. 10+816 pp.

DEANS (W. M.). See EWALD (P. P.).

DOUGALL (J.). See EWALD (P. P.).

DOWSE (C. M.). See HOLZER (W.).

EGGERT (J.). Einführung in die Röntgenphotographie. 6th edition. Leipzig, Hirzel, 1936. 217 pp.

ELSASSER (W. M.). See L'INSTITUT HENRI POINCARÉ.

EUCKEN (A.). See VON WEIZSÄCKER (C. F.).

EWALD (P. P.), PÖSCHL (T.), and PRANDTL (L.). The physics of solids and fluids. With recent developments. Authorized translation by J. Dougall and W. M. Deans. 2d edition. London, Glasgow, and Bombay, Blackie, 1936. 13+396 pp.

FISCHER (J.). Einführung in die klassische Elektrodynamik. Berlin, Springer, 1936. 8+199 pp.

FRENKEL (J.). Wave mechanics: elementary theory. (International Series of

Monographs on Physics.) 2d edition. London, Oxford University Press, 1936. 10+312 pp.

GORCEIX (C.). See BRARD (R.).

GRIMSEHL (E.). Lehrbuch der Physik. Edited by R. Tomaschek. Volume 1: Mechanik, Warmelehre, Akustik. 9th edition. Leipzig, Teubner, 1936. 7 +674 pp.

HAARMANN (E.). See AREND (J. P.).

HALDANE (J. B. S.). See JEANS (J.).

HARDY (W. B.). Collected scientific papers. London, Cambridge University Press, 1936. 12+922 pp.

HART (W. L.). Introduction to the mathematics of business. New York, Heath, 1936. 342 pp.

HILDEBRANDT (J. H.). Solubility of nonelectrolytes. 2d edition. New York, Reinhold, 1936. 203 pp.

HOLLMAN (H. E.). Physik und Technik der ultrakurzen Wellen. Volume 2: Die ultrakurzen Wellen in der Technik. Berlin, Springer, 1936. 8+306 pp.

HOLZER (W.) and WEISSENBERG (E.). Foundations of short wave therapy: physics—technics—indications; an introduction to the physico-technical principles and medical applications of short electric waves, for physicians and biologists. Physics and technics, by W. Holzer; Medical applications, by E. Weissenberg. Translated by J. Wilson and C. M. Dowse. London, Hutchinson's Scientific and Technical Publications, 1935. 228 pp.

HUXLEY (J.). See JEANS (J.).

L'INSTITUT HENRI POINCARÉ, Annales de. Volume 5, No. 3: Quelques propriétés typiques des corps solides, by R. Peierls; No. 4: La structure des noyaux atomiques complexes, by W. M. Elsasser. Paris, Les Presses Universitaires de France, 1935. 85 pp.

JEANS (J.), BRAGG (W.), APPLETON (E. V.), MELLANBY (E.), HALDANE (J. B. S.), and HUXLEY (J.). Scientific progress. New York, Macmillan, 1936. 210 pp.

JORDAN (P.). Die Physik des 20. Jahrhunderts. Braunschweig, Vieweg, 1936. 143 pp.

VON KRBEK (F.). Die Grundlagen der Quantenmechanik und ihre Mathematik. (Neue deutsche Forschungen, Abteilung Mathematik, volume 1.) Berlin, Junker and Dünnhaupt, 1936. 63 pp.

LABERENNE (P.). L'origine des mondes. Paris, Éditions Sociales Internationales, 1936. 320 pp.

LANDOLT—BÖRNSTEIN. Physikalische-chemische Tabellen. 5th revised edition, edited by W. A. Roth and K. Scheel. Supplementary volume 3. Part 3, 1st half. 16+537 pp. Part 3, 2d half. 686 pp. Berlin, Springer, 1936.

LEMON (H. B.). Cosmic rays thus far. London, Heinemann, 1936. 128 pp.

LEVY (H.) and ROTH (L.). Elements of probability. Oxford, Clarendon Press; London, Oxford University Press, 1936. 10+200 pp.

LOYARTE (R. G.). Fisica General. Volume 4. (Universidad Nacional de La Plata, Publicaciones de la Facultad de Ciencias Fisico-Matematicas.) La Plata, 1935. 8+547 pp.

MAHLER (G.) and MAHLER (K.). Physikalische Aufgabensammlung (Sammlung Göschen, No. 243.) Berlin, de Gruyter, 1936. 128 pp.

MAHLER (K.). See MAHLER (G.).

MASSÉ (P.). Hydrodynamique fluviale régimes variables. (Actualités Scientifiques et Industrielles, No. 280; Théories mécaniques (hydrodynamique—acoustique), V.) Paris, Hermann, 1935. 89 pp.

MELLANBY (E.). See JEANS (J.).

MOULTON (F. R.). Consider the heavens. New York, Doubleday Doran, 1935. 9+332 pp.

D'OCAGNE (M.). See VOLBERG (M.).

PEIERLS (R.). See L'INSTITUT HENRI POINCARÉ.

PHILIPP (K.). Kernspektren. (Hand- und Jahrbuch der chemischen Physik, volume 9, Section V.) Leipzig, Fock, 1937. 12+99 pp.

PIRENIAN (Z. M.). See SIMPSON (T. M.).

PLANCK (M.). Vom Wesen der Willensfreiheit. Leipzig, Barth, 1936. 30 pp.

PÖSCHL (T.). See EWALD (P. P.).

POIRÉE (J.). La mécanique à la portée de tous (cinématique, statique). Paris, Gauthier-Villars, 1936. 11+80 pp.

PRANDTL (L.). See EWALD (P. P.).

RETZOW (U.). Elektrotechnik und Witterung. Berlin, Springer, 1936. 121 pp.

ROTH (L.). See LEVY (H.).

ROTH (W. A.). See LANDOLT (—).

RUDOLPH (H.). Der Einfluss der Sonne auf den elektrischen und magnetischen Zustand der Erde. Leipzig, Hillmann, 1936. 61 pp.

SAGE (A.). Une science de l'ordre est cachée dans le monde des nombres. Paris, Libraire Nourry, 1936. 142 pp.

SCHEEL (K.). See LANDOLT (—).

SCHMIDT (E.). Einführung in die technische Thermodynamik. Berlin, Springer, 1936. 8+314 pp.

SIMPSON (T. M.), PIRENIAN (Z. M.), and CRENSHAW (B. H.). Mathematics of finance, 2d edition. New York, Prentice-Hall, 1936. 13+330+126 pp.

SMART (W. M.). Textbook on spherical astronomy. 2d edition. Cambridge, University Press, 1936. 12+430 pp.

TOMASCHEK (R.). See GRIMSEHL (E.).

VOLBERG (M.). Promenade récréative au pays d'Einstein. With a preface by M. d'Ocagne. Brussels, Librairie du Sphinx; Paris, Gauthier-Villars, 1936. 35 pp.

WEISSENBERG (E.). See HOLZER (W.).

VON WEIZSÄCKER (C. F.). Die Atomkerne. Grundlagen und Anwendungen ihrer Theorie. (Physik und Chemie und ihre Anwendungen in Einzeldarstellungen, volume 2.) Edited by A. Eucken and K. L. Wolf. Leipzig, Fock, 1937. 8+214 pp.

WERNERS (P.). Energieübertragung und -umwandlung mit Wechselstrom. Leipzig and Berlin, Teubner, 1935. 8+204 pp.

WILSON (J.). See HOLZER (W.).

WOLF (K. L.). See VON WEIZSÄCKER (C. F.).

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, February 20, 1937.

Abstracts must be in the hands of Secretary R. G. D. Richardson, 531 West 116th St., New York, N. Y., not later than January 23. Abstracts received by the secretaries on or before January 8 will appear in the January issue of this Bulletin. By invitation of the Program Committee, Professor T. R. Hollcroft will deliver an address entitled *The existence of algebraic plane curves with given singularities.*

NEW YORK CITY, March 26–27, 1937.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than March 5; they will appear in the March issue of this Bulletin. One session will be devoted to a symposium on Laplace integrals, with Professors Hille, Tamarkin, Widder, Bochner, Bohnenblust participating. A conference on algebraic geometry is being arranged; those interested should write to Professor Zariski.

STANFORD, CALIFORNIA, April 3, 1937.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, California Hall, Berkeley, California, not later than March 6; they will appear in the March issue of this Bulletin.

CHICAGO, ILLINOIS, April 9–10, 1937.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than March 6; they will appear in the March issue of this Bulletin. By invitation of the Program Committee, Professor W. T. Reid will deliver an address entitled *Boundary-value problems in the calculus of variations.*

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450–459 Ahnaip St., Menasha, Wis., or Low Memorial Library, 531 West 116th St., New York, N. Y.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1936, p. 67) are payable to the Treasurer of the Society, Professor P. A. Smith, Low Memorial Library, 531 West 116th St., New York, N. Y.

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